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Design of Multiproduct Batch Plants under Demand Uncertainty with Staged Capacity Expansions

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Abstract- This paper addresses the staged capacity expansion of multiproduct batch plants operating in single product campaign (SPC) mode and in the presence of product demand uncertainty. This extends previous work by the authors on the design problem. Staged capacity expansions are modeled as additions of extra units of equal size at each stage at the end of each time period. The normality assumption is employed for the uncertain product demands. The problem formulation involves an inner stage which identifies the optimum production levels and an outer stage which establishes the design/capacity expansion decisions by maximizing the net present value (NPV) of the plant. This two–stage stochastic optimization problem for capacity expansion is equivalently recast as a deterministic mixed–integer nonlinear programming (MINLP) problem. Finally, the proposed formulation and solution strategy are illustrated with an example problem.

INTRODUCTION

Batch plants provide a cost–effective alternative for the manufacture of specialty chemicals. At the design stage, the plant capacity must be set based on not only the present but also anticipated future product demands. Typically product demands increase over time. This implies that you have to either build a larger than necessary plant or systematically perform capacity expansions in the future. The last strategy provides a more cost effective way of meeting "just in time" future product demand. The challenge addressed here is to identify, at the design phase, the *location, size, and timing* of the capacity expansions given only probabilistic information about the future product demands.

A number of publications which address the problem of capacity expansion for continuous or batch plants with deterministic or uncertain product demands can be found in the literature. Wellons and Reklaitis (1989) proposed a conceptual formulation for batch plants design under uncertainty with staged capacity expansions. The authors suggested a distinction between "hard" and "soft" constraints and introduced penalty terms in the objective function for the latter type. Assuming that the demands change stepwise and the only uncertainty is in the time when the step change occurs, an analytical expression for the expected value of the objective function is derived which facilitates the solution of the model as an MINLP problem. Sahinidis and Grossmann (1991) proposed an MILP formulation for selecting capacity expansion policies for continuous chemical processes without explicitly considering product demand uncertainty. Based on a variable disaggregation technique efficient NLP reformulations of the MILP problem were proposed which quickly yield good suboptimal solutions. Berman et al (1994) suggested a scenario–based approach for capacity expansion in the service industries under product demand uncertainty. The capacity schedule specified the size, location, and timing of the expansions that maximized the expected profit. By utilizing a Lagrangian relaxation and exploiting the knapsack structure of the subproblems an efficient algorithmic procedure was proposed. Myers and Levary (1996) utilized linear programming to identify the best from several capacity expansion scenarios of a fuel additive production process. Ierapetritou and Pistikopoulos (1996) addressed the general problem of batch plant design under uncertainty. They developed a feasibility relaxation for the "soft" constraints and proposed a two–stage stochastic programming formulation. The latter is solved based on the discretization of the probability field through quadrature integration which leads to a single but typically large-scale nonconvex optimization problem. Based on the latter, a formulation which can accommodate staged capacity expansions was also proposed. Typically, the computational requirements for realistic models and problem sizes tend to be very large. The developments in this work are aimed at proposing a customized formulation and corresponding tractable solution strategy for staged capacity expansion of batch plants operating in (SPC) production mode with normally distributed future product demands.

PROBLEM DEFINITION

Given a set of uncertain product demands at different time periods, product recipe information, size factors and number of production stages, capacity ranges and maximum number of parallel processing units per stage, the problem addressed here is stated as follows:

Find the optimal design and capacity expansion policy of a multiproduct batch plant operating in SPC production mode such that the expected NPV of the batch plant, within a prespecified time horizon, is maximized while the production levels are optimally adjusted in response to product demand realizations and capacity expansions, in the form of additional units of equal size, occur at the end of each period.

The design objective, as stated above, suggests that a batch plant design and expansion policy is sought which establishes an optimal level of product demand satisfaction. This optimal level of demand satisfaction is established by striking the proper balance between profit from sales and investment costs. This balance is quantified through the NPV profitability measure and it is realized through the continuous optimal adjustment of the production policy of the batch plant given the current product demand profile. Product demands are modeled as normally distributed random variables. The plant is assumed to operate in SPC mode with overlapping operation. Transfer times from one unit to the next are assumed to be embedded in the processing times. The size factors, processing times, and profit margins are assumed to be independent of the capacity output and equipment sizes. Inventory transfers from one time period to the next are not considered due to their large length (typically more than one year). Equipment costs are assumed to be power functions of their capacities. Multiple units of equal capacity may operate in parallel at a particular stage to accommodate higher demands. Capacity expansion is modeled through the addition of parallel units of the same size and are allowed to occur only at the end of each period. Based on these assumptions the capacity expansion problem can be expressed as the following two–stage stochastic optimization problem:

$$\max_{V_j,B_i,N_{jt}} \quad E\left[\begin{array}{l} \max\limits_{Q_{it}} \sum\limits_{t=1}^{T}\sum\limits_{i=1}^{N} \delta_{R_t} P_{it} Q_{it} \\ Q_{it} \le \theta_{it},\ i=1,\ldots,N \\ \qquad\qquad t=1,\ldots,T \\ \sum\limits_{i=1}^{N}\left(\frac{Q_{it}}{B_i}\right) T_{L_{it}} \le H_t, R \\ \qquad\qquad t=1,\ldots,T \end{array}\right] - \sum_{t=1}^{T}\delta_{C_t}\sum_{j=1}^{M}\alpha_j (N_{jt}-N_{jt-1}) V_j^{\beta_j}$$

$$\text{st} \quad V_j \ge S_{ij} B_i,\ i=1,\ldots,N,\ j=1,\ldots,M$$

$$T_{L_{it}} = \max_{j=1,\ldots,M}\left\{\frac{t_{ij}}{N_{jt}}\right\},\ i=1,\ldots,N,\ t=1,\ldots,K$$

$$V_j^L \le V_j \le V_j^U,\ j=1,\ldots,M$$

$$N_j^L \le N_{jt} \le N_j^U,\ j=1,\ldots,M,\ t=1,\ldots,T$$

In the above formulation (see also Petkov and Maranas (1998)), the sets i,j, and t denote products, production stages and time periods, respectively. The parameters are the lengths of each time period H_t, the normally distributed uncertain product demands θ_{it}, the profit margins P_{it}, the coefficients $\delta_{R_t}, \delta_{C_t}$ which discount the future revenue and equipment costs to their present values, the preexponential cost coefficients α_j and exponents β_j, the size factors S_{ij}, and the processing times t_{ij}. The variables are the batch sizes B_i, the equipment capacities V_j, the number of parallel units N_{jt} at stage j and time period t, the production levels Q_{it}, and the cycle–time T_{Lit} for product i and period t.

The above formulation is a two–stage optimization problem. The inner problem sets the optimal operating policy that maximizes the profit for a given design and expansion policy, identified by V_j, B_i, N_{jt}, and realization of the uncertain demands θ_{it}. The first constraint of the inner problem disallows production levels to exceed product demands. The next one restricts the plant cycle–time to the available time horizon H_t of each time period. The second term in the objective of the outer problem measures the equipment cost as the discounted additive contribution of all the planned capacity expansions. The first constraint of the outer problem determines the minimum required equipment size at each stage. The second constraint identifies the rate limiting step for each product and time period accounting for the parallel units. Finally the last two constraints impose lower and upper bounds on equipment sizes and number of parallel units per stage and time period. The above formulation is partially convexified (apart from the horizon constraint) through the the following exponential transformation (Kocis and Grossmann (1988)):

$$V_j = \exp(v_j),\ B_i = \exp(b_i),$$

$$T_{Li} = \exp(t_{Li}),\ N_j = \exp(n_j)$$

$$n_j = \sum_{r=N_j^L}^{N_j^U} y_{jr}\ln(r) \text{ where } \sum_{r=N_j^L}^{N_j^U} y_{jr} = 1$$

Based on the developments described in detail in Petkov and Maranas (1998) a new approach for solving this problem is proposed. First, the inner optimization problem is solved analytically for the optimum production levels Q_{it} as a function of the demand realizations θ_{it} and the design variables (see Petkov and Maranas (1998)):

$$Q_{it}^{opt} = \begin{cases} \frac{1}{a_{i^t t}}\left(H_t - \sum_{\substack{i=1 \\ i \neq i^t}}^{N} a_{it}\theta_{it}\right) & \text{if } i = i^t \\ & \text{and } \sum_{i=1}^{N} a_{it}\theta_{it} \geq H_t \\ \theta_{it} & \text{otherwise} \end{cases}$$

where $a_{it} = \frac{T_{Lit}}{B_i}$ represents the amount of time it takes to produce a unit of product i in period t and $i^t = \arg\min_i \left(\frac{P_i}{a_{it}}\right)$ is the product with the smallest manufacturing profit per unit time at time period t. Note that by not imposing a lower bound of zero on the production levels Q_i a negative value for Q_{i^*} may occur. A more detail discussion can be found in Petkov and Maranas (1998).

In addition, by exploiting the normality assumption for the product demands, an analytical expression for the expected value of the optimum of the inner problem is also derived. The incorporation of these analytical results in the objective function after employing the exponential variable transformations of Kocis and Grossmann (1988) yields the following expressions:

$$\max \sum_{t=1}^{T}\sum_{i=1}^{N} \delta_{R_t} P_{it}\hat{\theta}_{it}$$
$$- \sum_{t=1}^{T} \delta_{R_t} \frac{P_{i^t t}}{a_{i^t t}} \sigma_{ct_t} \left[K_t \Phi(K_t) + f(K_t)\right]$$
$$- \sum_{t=1}^{T} \delta_{C_t} \sum_{j=1}^{M} \alpha_j \left[\exp\left(\beta_j v_j + \sum_{r=N_j^L}^{N_j^U} y_{jrt} \ln(r)\right) - \exp\left(\beta_j v_j + \sum_{r=N_j^L}^{N_j^U} y_{jrt-1} \ln(r)\right)\right]$$

$$\begin{aligned} \text{where } K_t &= \frac{\sum_{i=1}^{N} a_{it}\hat{\theta}_{it} - H_t}{\sigma_{ct_t}}, \; t = 1, \ldots, T \\ \sigma_{ct_t} &= \left[\sum_i a_{it}^2 \text{Var}(\theta_{it})\right]^{1/2}, \; t = 1, \ldots, T \\ a_{it} &= \exp\left(t_{L_{it}} - b_i\right), \; i = 1, \ldots, N, \\ & \qquad t = 1, \ldots, T, \\ i^t &= \arg\min_i \left(\frac{P_{it}}{a_{it}}\right), \; t = 1, \ldots, T \end{aligned}$$

The original two–stage stochastic programming problem is transformed into a single–stage deterministic MINLP optimization problem. Here f is the standardized normal probability density function and Φ the cumulative one. Also, σ_{ct_t} is the standard deviation of the cycle–time at period t and K_t measures the discrepancy between the required mean cycle–time and available horizon time scaled by its standard deviation. After (i) the introduction of binary variables for the selection of i^t, (ii) the subsequent exact linearization of the resulting bilinear terms, and (iii) the relaxation of some nonlinear equalities into equivalent convex inequalities (see Petkov and Maranas (1998) for details) the only remaining nonconvexities are the products $K_t\sigma_{ct_t}$ in the horizon constraint terms. By fixing the probabilities α_t of product demand satisfaction at each period the corresponding K_t variables are also set. This gives rise to a convex MINLP formulation for the capacity expansion problem for $\alpha_t \geq 0.5$. This observation motivates the solution strategy for the original nonconvex MINLP which can be thought of as a multiparametric convex MINLP problem with as many parameters as the number of periods. To illustrate the proposed developments a small example is next considered.

Illustrative Example

This example involves the design of a batch plant producing five different products whose expected lifetime is ten years. Each product recipe requires six production stages with up to five identical units per stage. The volumes of each processing unit can be between 500 and 3,000 liters. The data for processing times, size factors and cost and price data are given in Tables 1-3.

Table 1: Size factors S_{ij}(lit/kg)

	Stage					
Product	1	2	3	4	5	6
1	7.9	2.0	5.2	4.9	6.1	4.2
2	0.7	0.8	0.9	3.4	2.1	2.5
3	0.7	2.6	1.6	3.6	3.2	2.9
4	4.7	2.3	1.6	2.7	1.2	2.5
5	1.2	3.6	2.4	4.5	1.6	2.1

Table 2: Processing times t_{ij}(hours)

	Stage					
Product	1	2	3	4	5	6
1	6.4	4.7	8.3	3.9	2.1	1.2
2	6.8	6.4	6.5	4.4	2.3	3.2
3	1.0	6.3	5.4	11.9	5.7	6.2
4	3.2	3.0	3.5	3.3	2.8	3.4
5	2.1	2.5	4.2	3.6	3.7	2.2

Table 3: Equipment cost and profit margin data

Equipment cost coeff.			Price Data	
Stage	$\alpha_j(\$/lit)$	β_j	Product	$P_i(\$/kg)$
1	3000	0.6	1	3.5
2	3000	0.6	2	4.0
3	3000	0.6	3	3.0
4	3000	0.6	4	2.0
5	3000	0.6	5	4.5
6	3000	0.6		

The mean annual product demands for the first five years are 250, 150, 180, 160, and 120 tons, respectively, and the uncertainty in the product demands is assumed to be normally distributed with standard deviations equal to 10% of their respective mean values. In the last five years of the plant's lifetime the expected demand values are 20% higher and the uncertainty is represented by standard deviations equal to 20% of their new mean values. To account for the anticipated demand increase in the second five-year period, one plant expansion is planned for five years after the plant starts operation. This problem description implies that there are two five–year periods. This leads to an MINLP with 10 binary variables identifying the (i_t)'s, 60 binary variables modeling the number units per stage (6 stages × up to 5 units × two five-year periods), 1049 continuous variables, and 295 constraints.

The problem is iteratively solved for fixed values of K_t, $t = 1, 2$ corresponding to probabilities α of meeting all product demands between 0.5 and 0.95. Each one of these problems is a convex MINLP and is solved using the DICOPT/GAMS interface. The expected NPV values are plotted as a function of the probability levels in Figure 1.

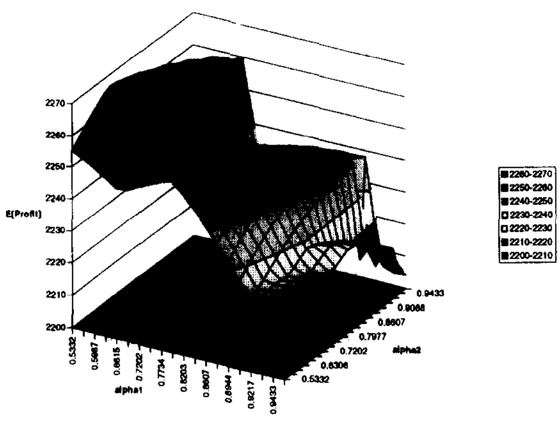

Figure 1: Expected NPV versus product demand satisfaction probability levels for t=1,2.

This plot reveals the "complex" relation between the optimum expected NPV values and the probability levels. The highly varying nature of the surface plot manifests considerable changes in the optimal batch plant design for different levels of α_t. Note that the average slope magnitude of the surface plot in the direction of α_2 is much greater than in the direction of α_1. This is a result of the higher level of uncertainty associated with the product demands in the second five-year period. The expected NPV is maximized for $\alpha_1 = 0.82$ and $\alpha_2 = 0.62$ assuming a value of \$2,267×10^3. The optimal equipment capacities for the six stages are 2910, 1481, 1915, 2042, 2247 and 1645 respectively. For the first five years, stages 1 and 2 have two units and stages 3,4 and 5 have 3,4 and 1 units respectively. To optimally accommodate the higher product demand in the second five-year periods, one additional unit is added at stages 5 and 6. Note that if no plant expansion was allowed, the maximum expected NPV value would have been \$2,168×10^3 (about 5% less).

SUMMARY

This paper extended previous work of Petkov and Maranas (1998) on the design of multiproduct batch plants under demand uncertainty to account for staged capacity expansions. The resulting formulation is a T–parameter convex MINLP for product demand satisfaction probabilities which are higher than 50%. The K_t variables are revealed to be the only source of nonconvexities. This provides an avenue for the construction of an efficient global optimization approach by selectively branching on only the K_t variables.

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