Midterm Supply Chain Planning Under Demand Uncertainty : Customer Demand Satisfaction and Inventory Management

Anshuman Gupta and Costas D. Maranas[1]
Department of Chemical Engineering, The Pennsylvania State University
University Park, PA 16802

Conor M. McDonald
Dupont Lycra®
Barley Mill Plaza, BMP25-2361
Wilmington, Delaware 19880-0025

[1] Author to whom all correspondence should be addressed. E-mail:costas@psu.edu. Phone:814-863-9958. Fax: 814-865-7846

Abstract

This paper utilizes the framework of midterm, multisite supply chain planning under demand uncertainty (Gupta and Maranas, 2000) to safeguard against inventory depletion at the production sites and excessive shortage at the customer. A chance constraint programming approach in conjunction with a two-stage stochastic programming methodology is utilized for capturing the trade-off between *customer demand satisfaction (CDS)* and production costs. In the proposed model, the production decisions are made before demand realization while the supply chain decisions are delayed. The challenge associated with obtaining the second stage recourse function is resolved by first obtaining a closed-form solution of the inner optimization problem using linear programming duality followed by expectation evaluation by analytical integration. In addition, analytical expressions for the mean and standard deviation of the inventory are derived and used for setting the appropriate CDS level in the supply chain. A three-site example supply chain is studied within the proposed framework for providing quantitative guidelines for setting customer satisfaction levels and uncovering effective inventory management options. Results indicate that significant improvement in guaranteed service levels can be obtained for a small increase in the total cost.

Introduction

Product demand variability can be identified as one of the key sources of uncertainty in any supply chain. Failure to account for significant product demand fluctuations in the medium term (1-2 years) by deterministic planning models may either lead to excessively high production costs (translating to high inventory charges) or unsatisfied customer demand and loss of market share. Recognition of this fact has motivated significant work aimed at studying process planning and scheduling under demand uncertainty. Specifically, most of the research on this problem has largely focussed on short term scheduling of batch plants (Shah and Pantelides, 1992; Ierapetritou and Pistikopoulos, 1996; Petkov and Maranas, 1998) and long term capacity planning of chemical processes (Clay and Grossmann, 1994; Liu and Sahinidis, 1996). Some of the important features that have not been considered in great detail include (semi)continuous processes, multisite supply chains and midterm planning time frames. In view of this, incorporation of demand uncertainty in midterm planning of multisite supply chains having (semi)continuous processing attributes is discussed in Gupta and Maranas (2000) through a two-stage stochastic programming framework. In this paper, the trade-off involved between inventory depletion and production costs in the face of uncertainty is captured in a probabilistic framework through chance-constraints. A customized solution technique, aimed at reducing the computational expense typically associated with stochastic optimization problems, is developed. The basic idea of the proposed methodology consists of translating the stochastic attributes of the problem into an equivalent deterministic form which can be handled efficiently.

The rest of the paper is organized as follows. In the next section, the two-stage stochastic model, which is based on the deterministic midterm planning model of McDonald and Karimi (1997), is presented. Next, the key elements of the proposed solution methodology are discussed briefly. The details of the analysis can be found in Gupta and Maranas (2000). The main part of the paper proposes a chance-constraint based approach for capturing the trade-off between customer shortage and production costs. The other critical issue of excessive inventory depletion is then addressed within the proposed framework by obtaining analytical expressions for the mean and standard deviation of the inventory. These are utilized for setting the appropriate service level in the supply chain. Computational results for an example supply chain are presented followed by concluding comments.

Two-Stage Model

The slot-type economic lot sizing model of McDonald and Karimi (1997) is adopted as the benchmark formulation for this work. The variables of this model can be partitioned into two categories (Gupta and Maranas, 2000) based on whether the corresponding tasks need to be carried out before or after demand realization. The *production variables* model activities such as raw material consumption, capacity utilization and final product production. Due to the significant lead times associated with these tasks, they are modeled as "here-and-now" decisions which need to be taken prior to demand realization. Post production activities such as inventory management and supply of finished product to customer, on the other hand, can be performed much faster. Consequently, these constitute the *supply-chain variables* which can be fine-tuned in a "wait-and-see" setting after realization of the actual demand. This classification of variables naturally extends to the constraints of the problem (Gupta and Maranas, 2000) and results in the following two-stage formulation.

$$\min_{\substack{P_{ijs}, RL_{ijs}, FRL_{fjs} \\ A_{is}, C_{is}, W_{iss'} \geq 0 \\ Y_{fjs} \in \{0,1\}}} \sum_{f,j,s} FC_{fs} Y_{fjs} + \sum_{i,j,s} \nu_{ijs} P_{ijs} + \sum_{i,s} p_{is} C_{is} + \sum_{i,s,s'} t_{iss'} W_{iss'} + Q$$

$$Q = E_{\theta_i} \left[\begin{array}{ll} \min\limits_{S_{is}, I_{is}, I_{is}^{\Delta}, I_{is}^{-} \geq 0} & \sum\limits_{i,s} t_{is} S_{is} + \sum\limits_{i,s} h_{is} I_{is} + \sum\limits_{i,s} \zeta_{is} I_{is}^{\Delta} + \sum\limits_{i} \mu_i I_i^{-} \\ & \sum\limits_{s} S_{is} \leq \theta_i \\ & I_{is} = A_{is} - S_{is} \\ & \theta_i - \sum\limits_{s} S_{is} \leq I_i^{-} \leq \theta_i \\ & I_{is}^{L} - I_{is} \leq I_{is}^{\Delta} \leq I_{is}^{L} \end{array} \right]$$

subject to

$$P_{ijs} = R_{ijs} RL_{ijs}$$

$$C_{is} = \sum_{i'} \beta_{i'is} \sum_{j} P_{i'js} = \sum_{s'} W_{is's}$$

$$A_{is} = I_{is}^{0} + \sum_{j} P_{ijs} - \sum_{s'} W_{iss'}$$

$$\sum_{f} \sum_{i:\lambda_{if}=1} RL_{ijs} \leq H_{fjs}$$

$$MRL_{fjs} Y_{fjs} \leq \sum_{i:\lambda_{if}=1} RL_{ijs} \leq H_{fjs} Y_{fjs}$$

In the above formulation, the various indices are as follows: *i*(products), *f* (product families), *j* (processing equipment) and *s* (production sites). The various cost parameters are FC_{fs} (setup cost), ν_{ijs} (variable production cost), p_{is} (raw material cost), $t_{iss'}$(intersite transportation cost), t_{is} (customer-site transportation cost), h_{is} (inventory holding cost), ζ_{is} (safety stock violation penalty) and μ_i (lost revenue cost). MRL_{fjs} is the minimum runlength, H_{fjs} is the total available processing time while R_{ijs} is the rate of production and $\beta_{i'is}$ is the material balance coefficient. Other parameters are I^0_{is} (initial inventory), I^L_{is} (safety stock level) and θ_i (uncertain demand).

The objective function of the above formulation is composed of two terms. The first term, subject to the outer optimization problem constraints, accounts for the costs incurred in the production stage. Production stage variables include A_{is} (availability), P_{ijs} (production amount), RL_{ijs} (runlength), C_{is} (raw material consumption), $W_{is's}$ (intersite shipment) and Y_{fjs} (setup). The second term Q is obtained by applying the expectation operator to the optimal value of an embedded optimization problem. The constraints of this inner *recourse* problem are the supply chain constraints and the inner stage variables, S_{is} (supply), I_{is} (inventory), I_{is}^{Δ} (safety stock deficit) and I_{is}^{-} (customer shortage), are the supply chain variables. The interaction between the outer (production) and inner (supply chain) problems takes place through the inventory balance constraint which forces the inventory to be equal to the difference between the amount available for supply (A_{is}) and the actual supply to the customer (S_{is}). The basic idea of the proposed methodology consists of obtaining a closed-form analytical expression for Q (the recourse function) in terms of the first stage production variables (specifically A_{is}). This is achieved by first explicitly solving the inner recourse problem followed by analytical expectation evaluation (Gupta and Maranas, 2000).

Optimal Supply Policies

The inner supply chain planning problem is solved analytically using linear programming (LP) duality (Gupta and Maranas, 2000). In particular, the key principle utilized is the strong LP duality theorem. To aid the developments, some additional notation is introduced. The production sites are classified as either *internally sufficient* (IS) or *internally deficient* (ID) as

$$\text{IS} = \left\{ s \in S \mid A_{is} - I_{is}^{L} \geq 0 \right\} \text{ and } \text{ID} = \left\{ s \in S \mid A_{is} - I_{is}^{L} \leq 0 \right\}$$

Thus, at the ID sites, safety stock violation cannot be avoided as the amount available for supply is not adequate to meet the safety stock requirement. Two additional cost parameters, the *over-safety stock supply cost* (γ_{is}) and the *under-safety stock supply cost* (ω_{is}), are introduced. These are given by

$$\gamma_{is} = t_{is} - h_{is} \text{ and } \omega_{is} = t_{is} - h_{is} + \zeta_{is}$$

These represent per unit costs of shipping a product from above (γ_{is}) or below (ω_{is}) the safety stock level. Ranking of the IS and/or ID sites on the basis of these cost parameters forms the key principle of the proposed methodology. By exploiting the network representation of the inner supply chain planning problem, three distinct demand regimes are uncovered in the optimal solution. These are referred to as regimes of *low*, *intermediate* and *high* demand realizations. A summary of the supply policies for these demand regimes is provided below.

Low Demand Regime

In this demand regime, the safety stock violation penalty is restricted to the ID sites by directing all customer product supply from the IS sites. Ranking of the IS sites in increasing order of over-safety stock supply cost (γ_{is}) represents the optimal sequence in which the sites service the customer. Therefore, for an IS site s_l^* with

$$\sum_{\substack{s=1 \\ s \in IS}}^{s_l^*-1} \left(A_{is} - I_{is}^{L} \right) \leq \theta_i \leq \sum_{\substack{s=1 \\ s \in IS}}^{s_l^*} \left(A_{is} - I_{is}^{L} \right)$$

the optimal supply policies are given by

$$S_{is} = \begin{cases} A_{is} - I_{is}^{L} & \forall\ s \le s_l^* - 1, s \in IS \\ \theta_i - \sum_{\substack{s=1 \\ s \in IS}}^{s_l^*-1} \left(A_{is} - I_{is}^{L} \right) & \forall\ s = s_l^*, s \in IS \\ 0 & \forall\ s \ge s_l^* + 1, s \in IS \text{ and } \forall\ s \in ID \end{cases}$$

Note that given the optimal supply policies, the optimal values for the remaining supply chain variables can be calculated (for any demand regime) as:

$$I_{is} = A_{is} - S_{is}\ ;\ I_{is}^{\Delta} = \max\left(0, I_{is}^{L} - I_{is}\right);\ I_i^{-} = \max\left(0, \theta_i - \sum_{s} S_{is}\right)$$

No safety stock violations occur at any of the IS sites and no sales are lost in this demand regime. The transition to the intermediate demand regime occurs at a demand realization given by $\Theta_i^{L \to I} = \sum_{s \in IS} \left(A_{is} - I_{is}^{L} \right)$

Intermediate Demand Regime

In the intermediate demand regime, the entire customer demand is met only at the expense of incurring safety stock violation penalties at some or all of the production sites. Consequently, all sites are ranked in increasing order of under-safety stock supply cost (ω_{is}) irrespective of their type (i.e., IS or ID). This establishes the order in which demand is allocated to the production sites in the intermediate demand regime. For a site s_i^* defined as

$$\Theta_i^{L \to I} + \sum_{\substack{s=1 \\ s \in ID}}^{s_i^*-1} A_{is} + \sum_{\substack{s=1 \\ s \in IS}}^{s_i^*-1} I_{is}^{L} \le \theta_i \le \Theta_i^{L \to I} + \sum_{\substack{s=1 \\ s \in ID}}^{s_i^*} A_{is} + \sum_{\substack{s=1 \\ s \in IS}}^{s_i^*} I_{is}^{L}$$

the corresponding optimal supply policies are

$$S_{is} = \begin{cases} A_{is} & \forall\ s \le s_i^* - 1 \\ \theta_i - \sum_{s=1}^{s_i^*-1} A_{is} - \sum_{\substack{s \ge s_i^*+1 \\ s \in IS}} \left(A_{is} - I_{is}^{L} \right) & \forall\ s = s_i^* \\ 0 & \forall\ s \ge s_i^* + 1, s \in ID \\ A_{is} - I_{is}^{L} & \forall\ s \ge s_i^* + 1, s \in IS \end{cases}$$

As in the low demand regime, no shortfall occurs at the customer. Transition to the high demand regime occurs at the demand realization given by $\Theta_i^{I \to H} = \sum_{s} A_{is}$

High Demand Regime

In this demand regime, due to high levels of customer demand, the entire amount available for supply is shipped to the customer following the "customer priority" paradigm (Gupta and Maranas, 2000). Therefore, for $\theta_i \geq \sum_s A_{is}$ the optimal supply policies are given by

$$S_{is} = A_{is} \qquad \forall s$$

Inventories at *all* the sites are completely depleted resulting in maximum safety stock violation charges. The high demand regime is unique in the sense that it is the *only* regime in which unsatisfied demand occurs at the customer and the inventory at *all* the sites is zero.

After solving the inner problem explicitly, the recourse function *Q* is calculated by integrating the optimal objective value over all possible demand realizations. This undertaking translates into the calculation of three conditional probability integrals, one for each of the three demand regimes. Consequently, by invoking the normality assumption for the product demands followed by analytical integration, the following form for the recourse function is obtained (for details see Gupta and Maranas, 2000).

$$Q = \sum_{i,s} a_{is} K_{is} + \sum_{i,s} b_{is} \sigma_i \left[K_{is} \Phi(K_{is}) + f(K_{is}) \right]$$

where a_{is}, b_{is} are constants which are functions of the second stage cost parameters, σ_i is the standard deviation of the demand, *f( )* and *Φ( )* are the normal density and cumulative distribution functions respectively and

$$K_{is} = \frac{\left(A_{is} - \theta_i^m\right)}{\sigma_i}$$

with θ_i^m as the mean demand.

Customer Demand Satisfaction

Missed sales in the high demand regime are unacceptable from a customer relationship perspective given the constant shifting of customer loyalties in today's highly competitive business environment. Therefore, to safeguard against this scenario in a probabilistic framework, the following chance constraint is introduced.

$$\Pr\left[\theta_i \leq \Theta_i^{I \to H} = \sum_s A_{is} \right] \geq \alpha$$

where α is the target *customer demand satisfaction (CDS) level*. This constraint ensures that the probability of operating the supply chain in the high demand regime is less than (1-α). By changing the value of α, optimal trade-off curves can be constructed between total cost and frequency of missed customer demand. The deterministic equivalent form for the chance constraint is obtained as

$$\sum_s A_{is} \geq \theta_i^m + \sigma_i \Phi^{-1}(\alpha)$$

where $\Phi^{-1}(\ )$ is the inverse normal cumulative distribution function. The stochastic attributes of the original problem are thus transformed into (exact) equivalent deterministic form resulting in a convex nonlinear mixed integer programming problem (Gupta and Maranas, 2000).

Inventory Control

In addition to lost sales, the high demand regime is also characterized by depletion of inventory in the entire supply chain for a particular product. This could pose significant operational challenges. Even though the chance constraint introduced to limit customer shortage favorably affects the inventory profiles in the supply chain by increasing the amount available for supply, the problem of inventory depletion at a production site is not completely resolved. Excessively low inventory levels at individual sites might still occur as the chance constraint only relates the *aggregate* amount available in the supply chain to the CDS level. Consequently, a more robust operation of the supply chain from an inventory management perspective can be achieved by studying the variation of the probability distribution and the corresponding mean and standard deviation of the inventory level with changing CDS targets. For this undertaking, analytical expressions relating the expected level and standard deviation of the inventory to the amount available for supply at each production site are developed. By applying the expectation operator to the inventory balance constraint, the average inventory can be related to the expected supply as

$$E_{\theta_i}[I_{is}] = A_{is} - E_{\theta_i}[S_{is}]$$

By conditionally integrating the supply policies presented in the previous section and substituting in the above equation, the mean inventory is obtained as

$$\begin{aligned} E_{\theta_i}[I_{is}] = {} & \sigma_i\left[K1_{i,s=r_s^L}\,\Phi\left(K1_{i,s=r_s^L}\right) + f\left(K1_{i,s=r_s^L}\right)\right] \\ & + \sigma_i\left[K2_{i,s=r_s^I}\,\Phi\left(K2_{i,s=r_s^I}\right) + f\left(K2_{i,s=r_s^I}\right)\right] \\ & - \sigma_i\left[K1_{i,s=r_s^L-1}\,\Phi\left(K1_{i,s=r_s^L-1}\right) + f\left(K1_{i,s=r_s^L-1}\right)\right] \\ & - \sigma_i\left[K2_{i,s=r_s^I-1}\,\Phi\left(K2_{i,s=r_s^I-1}\right) + f\left(K2_{i,s=r_s^I-1}\right)\right] \end{aligned}$$

where

$$K1_{is} = \frac{\displaystyle\sum_{\substack{s=1 \\ s\in IS}}^{s_i^*}\left(A_{is} - I_{is}^L\right) - \theta_i^m}{\sigma_i}$$

$$K2_{is} = \frac{\Theta_i^{L\to I} + \displaystyle\sum_{\substack{s=1 \\ s\in ID}}^{s_i^*} A_{is} + \sum_{\substack{s=1 \\ s\in IS}}^{s_i^*} I_{is}^L - \theta_i^m}{\sigma_i}$$

and r_s^L and r_s^I are the ranks of site s in the low and intermediate demand regimes respectively. An interesting observation can be made based on the expression for the mean inventory. Convexity of the $K\Phi(K) + f(K)$ terms implies that the mean inventory is the sum of convex (first two) and concave (last two) terms. In the light of this observation, *oscillations* in the expected inventory level due to the change in relative magnitudes of these two components, might be expected. This trend is actually observed in the supply chain example studied later in the paper.

In addition to the expected inventory, the standard deviation of the inventory distribution can also be calculated within the analytical framework. By squaring the inventory balance constraint followed by conditional expectation evaluation, the following expression for the inventory standard deviation is obtained.

$$SD[I_{is}] = \sqrt{Var[I_{is}]} = \sqrt{E_{\theta_i}[I_{is}^2] - E_{\theta_i}[I_{is}]^2}$$

where

$$\begin{aligned}
E_{\theta_i}[I_{is}^2] = {} & \Phi\left(K1_{i,s=r_s^L-1}\right)A_{is}^2 \\
& + \left[\Phi\left(K1_{i,s=r_s^L}\right) - \Phi\left(K1_{i,s=r_s^L-1}\right)\right]\left[\left(A_{is} + \sigma_i K1_{i,s=r_s^L-1}\right)^2 + \sigma_i^2\right] \\
& - \sigma_i^2\left[K1_{i,s=r_s^L} f\left(K1_{i,s=r_s^L}\right) - K1_{i,s=r_s^L-1} f\left(K1_{i,s=r_s^L-1}\right)\right] \\
& + 2\sigma_i\left[A_{is} + \sigma_i K1_{i,s=r_s^L-1}\right]\left[f\left(K1_{i,s=r_s^L}\right) - f\left(K1_{i,s=r_s^L-1}\right)\right] \\
& + \left[\Phi\left(K2_{i,s=r_s^I-1}\right) - \Phi\left(K1_{i,s=r_s^L}\right)\right]\left(I_{is}^L\right)^2 \\
& + \left[\Phi\left(K2_{i,s=r_s^I}\right) - \Phi\left(K1_{i,s=r_s^I-1}\right)\right]\left[\left(I_{is}^L + \sigma_i K2_{i,s=r_s^I-1}\right)^2 + \sigma_i^2\right] \\
& - \sigma_i^2\left[K2_{i,s=r_s^I} f\left(K2_{i,s=r_s^I}\right) - K2_{i,s=r_s^I-1} f\left(K2_{i,s=r_s^I-1}\right)\right] \\
& + 2\sigma_i\left[I_{is}^L + \sigma_i K2_{i,s=r_s^I-1}\right]\left[f\left(K2_{i,s=r_s^I}\right) - f\left(K2_{i,s=r_s^I-1}\right)\right]
\end{aligned}$$

The above result is based on the following conditional expectation results.

$$E_z\left[z^2 \middle| \quad z \le K1\right] = 1 - K1\frac{f(K1)}{\Phi(K1)}$$

$$E_z\left[z^2 \middle| \quad K1 \le z \le K2\right] = 1 - \frac{\left(K2\,f(K2) - K1\,f(K1)\right)}{\left(\Phi(K2) - \Phi(K1)\right)}$$

where z follows a standard normal distribution N(0,1). By studying the variation in the mean and standard deviation of inventory levels at the various production sites with changing CDS levels, the choice of the appropriate service level can be further refined to account for inventory depletion.

Example

The proposed methodology is highlighted through a three-site example supply chain illustrated in Figure 1. A total of 10 products, grouped into 5 product families, are manufactured at these facilities, which are characterized by different processing and cost attributes. Each site has single processing equipment dedicated for each product family and the products within the family compete for the limited capacity of this equipment. Setup charges are incurred for each production campaign and product demand exists at a single customer. The data for the example is given in Tables 1 through 3.

First, the deterministic midterm planning problem is solved. This yields an optimal solution of 4,165. Subsequently, the deterministic equivalent problem under uncertainty is solved *without* enforcing the chance constraints. Solution of this convex MINLP using DICOPT accessed via GAMS, results in an optimal expected cost of 4,726 obtained in 43 CPU seconds. The model consists of 301 constraints, 286 continuous variables and 75 discrete variables and is solved to optimality in 9 iterations. The difference between the deterministic and stochastic solutions quantifies the impact of uncertainty in the supply chain. The values obtained are verified, at considerably higher computational expense, by solving the same problem instance with Monte Carlo (MC) sampling. Comparison of computational requirements (2038 CPU seconds for MC with 500 sampled scenarios as compared to 43 CPU seconds) highlights the efficiency of the proposed methodology.

Customer Demand Satisfaction

Given this “base” production setting for the supply chain, the “base” CDS level is calculated for each product (Figure 2). This represents the probability that the demand realized for a particular product lies either in the low or the intermediate demand regime. Equivalently, it is the probability that no customer orders are lost. As the figure indicates, CDS levels ranging from 70% to 80% are achieved with the base production plan. Next, the chance constraint is introduced into the problem and the problem is solved for varying CDS target levels. The optimal total costs thus incurred are shown in Figure 3. As illustrated in the figure, the total cost increases relatively linearly with the CDS level. This initial linear relation, however, changes to an exponential one at CDS levels ranging from 90-97%. This implies that at the expense of modest cost increase, the customer demand satisfaction can be improved to about 90-97%. Also, the continuously increasing slope of the curve implies that the cost incurred per unit change in CDS level increases with the CDS level. This is expected in the light of the classic law of diminishing returns.

To gain further insight into the operation of the supply chain with respect to varying levels of CDS, the total cost incurred is analyzed in terms of its deterministic (first stage production costs) and stochastic (second stage supply costs) components. The resulting trade-off curve obtained is shown in Figure 4. As the CDS target level is increased, the expected supply chain costs decrease as the largest component of this cost, the lost revenue, is reduced. The production costs, on the other hand, increase primarily because of the additional setups required for increasing the amount available for supply. For a unit increase in the production costs, the reduction in the supply chain costs is approximately 60%. This is in agreement with the observation that the total cost increases with increasing CDS level (Figure 3). The difference between the additional production costs and the resulting supply chain savings can be viewed as the cost incurred for making the supply chain more robust and reliable from a customer service viewpoint. As shown in Figure 4, the expected supply chain savings level off in the range of 95-97% CDS level, which is approximately the level at which the total cost starts increasing exponentially (Figure 3).

Inventory Control

Having addressed customer shortage management through the chance constraint, the issue of inventory control in the supply chain is considered. The variations of the probability distributions, corresponding expected values and standard deviations of the inventory of product 1, with changing CDS levels are shown in Figure 5 through Figure 7. At site 1, the probability of having the inventory at the safety stock level of 10 units is relatively high even for low CDS levels (Figure 5(a)). This probability increases with increasing CDS level as more product is made available for supply. The expected inventory correspondingly increases with the CDS level while the standard deviation decreases as shown in Figure 5(b). Thus, with respect to inventory management considerations at site 1, a high CDS level would be preferred as this would translate into high levels of inventory with lower variability. The inventory distribution at site 2 is considerably more depleted than in site 1 (Figure 6(a)) as the probability of having low inventory is relatively high even at high CDS levels. For instance, there is approximately 50% probability of completely depleting inventory at a CDS level as high as 99%. However, the expected inventory profile at site 2 in Figure 6(b) indicates average inventory levels in the range of 6-7 units between CDS levels of 70-80%. These values are misleading when viewed in light of the actual probability distribution of the inventory. They can be attributed to extremely high inventory levels (30-40 units) existing at very low probability levels (0.05-0.1) at site 2. This is also reflected in the high standard deviation (8-9 units) of the inventory level in this CDS level range. Almost complete inventory depletion is predicted at site 2 in CDS levels ranging from 80-90%. This trend is followed by significant cyclical variations in expected inventory with increasing CDS level coupled with correlated variations in the corresponding standard deviation. These oscillations can be attributed to the changing relative magnitudes of the convex and concave terms in the expression for the expected inventory. At site 3, complete inventory depletion can be predicted with 100% probability for CDS levels ranging from 70% to 80% as shown in Figure 7(a). This observation is also supported by the expected inventory profile in Figure 7(b). Average inventory of 8-9 units with a corresponding standard deviation of 8-9 units is expected to occur between 80-90% CDS level. The high standard deviation is also indicated by the probability distribution in this range of CDS values as most of the inventory levels that can occur are approximately equally likely. Cyclical changes similar to site 2 are also observed at site 3 at higher CDS levels.

Using these inventory profiles for the three sites, the choice of the “optimal” CDS level at which the supply chain should operate can be determined. The CDS level range of 90-97% as determined by the service level considerations can be further refined to effectively account for inventory control issues. Based on the results indicated in Figures 5 through 7, an appealing CDS level to operate the supply chain at is 97%. The expected inventories at sites 1, 2 and 3 at this CDS level are approximately 9 units, 6 units and 13 units respectively. The corresponding supply chain and production costs are 2,041 and 3,011 respectively (5,052 total cost). Therefore, an improvement of 17% in the CDS level is achieved over the base setting at the expense of 7% additional cost.

Hedging Inventory Risk

An interesting observation that can be made by comparing the inventory profiles at site 2 (Figure 6(b)) and site 3 (Figure 7(b)) is that the inventory variations are complementary at the two sites. Low expected inventory at one site corresponds to high levels at the other. This trend is also incorporated in the cyclical fluctuations in the high CDS range where the oscillations at the two sites are “out-of-phase’’. Similar trends are also seen for the standard deviation of the inventory. This observation can be potentially utilized for modifying the risk profile of the inventory in the supply chain. By considering the option of integrating the manufacturing capacity of product 1 at sites 2 and 3, the inventory risk as characterized by the standard deviation can be effectively “squeezed’’ out from the system. The corresponding hedged position is shown in Figure 8. Smoother inventory profiles in conjunction with relatively constant standard deviation can make the operation of the supply chain more robust.

Summary

In this paper, a two-stage modeling framework coupled with a chance constraint programming approach was utilized for incorporating demand uncertainty and issues of customer demand satisfaction and inventory management. In addition, inventory depletion in multisite supply chains was addressed within the proposed analytical framework. The customized solution procedure for the two-stage problem (Gupta and Maranas, 2000) was extended to account for the probabilistic constraints introduced to enforce desired customer demand satisfaction levels. Analytical expressions for the mean and standard deviation of the inventory levels at the various production sites were used for making the supply chain more robust from an inventory management perspective. The fact that significant improvements in terms of guaranteed service levels to the customer could be achieved at relatively small additional cost was indicated through an example supply chain planning problem. The possibility of uncovering potential strategic options for managing inventory risk in the supply chain was also highlighted.

Acknowledgements

The authors gratefully acknowledge financial support by the NSF-GOALI Grant CTS-9907123 and DuPont Educational Aid Grant 1999/2000.

References

Clay, R. L. and Grossmann, I. E., Optimization of stochastic planning models. *Trans. IchemE*, **72**, 415 (1994).

Gupta, A. and Maranas, C. D., A two-stage modeling and solution framework for multisite midterm planning under demand uncertainty. *Accepted for publication in I&EC Research* (2000).

Ierapetritou, M. G. and Pistikopoulos, E. N., Batch plant design and operations under uncertainty. *I&EC Research*, **35**, 772 (1996).

Liu, M. L. and Sahinidis, N. V., Optimization in process planning under uncertainty. *I&EC Research,* **35,** 4154 (1996).

McDonald, C. and Karimi, I. A., Planning and scheduling of parallel semicontinuous processes. 1. Production planning. *I&EC Research*, **36,** 2691 (1997).

Petkov, S. B. and Maranas, C. D., Design of single-product campaign batch plants under demand uncertainty. *AIChE Journal,* **44,** 896 (1998).

Shah, N. and Pantelides, C. C., Design of multipurpose batch plants with uncertain production requirements. *I&EC Research*, **31,** 1325 (1992).

Table 1. Family dependent production parameters for the example supply chain

Site(s)	Product Family (f)	Fixed Cost (FC_{fs})	Minimum Runlength (MRL_{fs})	Available Time (H_{fs})
1	{1,2,3,4,5}	{4.5,4.8,5.5,6.2,4.5}	{75,75,75,75,75}	{150,250,200,185,190}
2	{1,2,3,4,5}	{6.5,3.5,6.5,4.5,6.5}	{75,75,75,75,75}	{250,200,150,200,190}
3	{1,2,3,4,5}	{6.5,5.2,5.1,4.7,6.5}	{50,50,50,50,50}	{225,270,250,150,265}

Table 2. Cost parameters for the example problem

Parameter	s=1	s=2	s=3
Production Cost (ν_{is})	2.5	2.3	2.6
Transportation Cost (t_{is})	1.1	1.2	1.3
Inventory Cost (h_{is})	1.8	1.7	1.6
Underpenalty Cost (ζ_{is})	2.7	2.3	2.2
Production Rate (R_{is})	0.5	0.6	0.5
Initial Inventory (I^0_{is})	0	0	0
Safety Stock Level (I^L_{is})	10	15	25

Table 3. Demand distributions and revenues for the products

Product (i)	Demand (θ_i)	Revenue (μ_i)
1	N(70,15)	10.0
2	N(50,10)	9.0
3	N(85,10)	9.5
4	N(100,30)	10.5
5	N(100,20)	9.0
6	N(90,20)	11.0
7	N(55,15)	10.8
8	N(80,20)	10.0
9	N(95,30)	12.0
10	N(110,25)	8.5

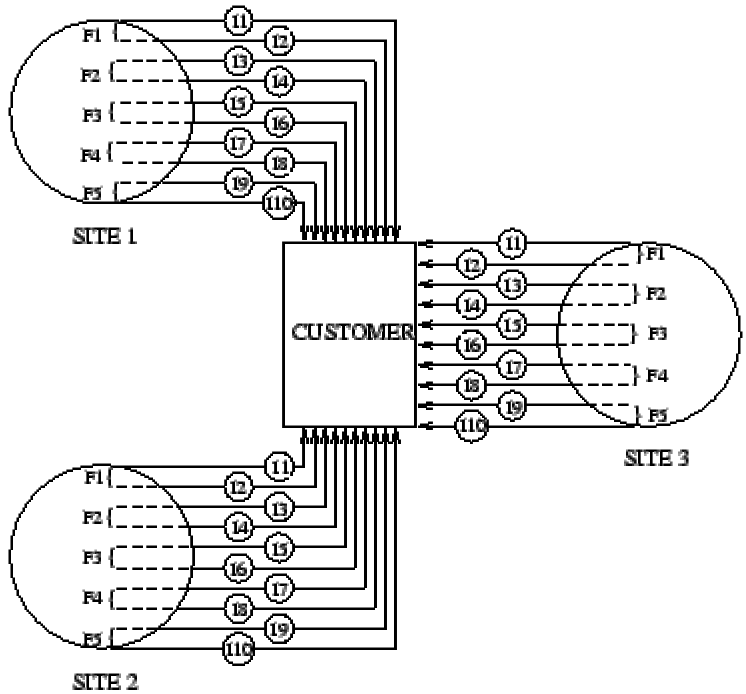


Figure 1. Three-site example supply chain

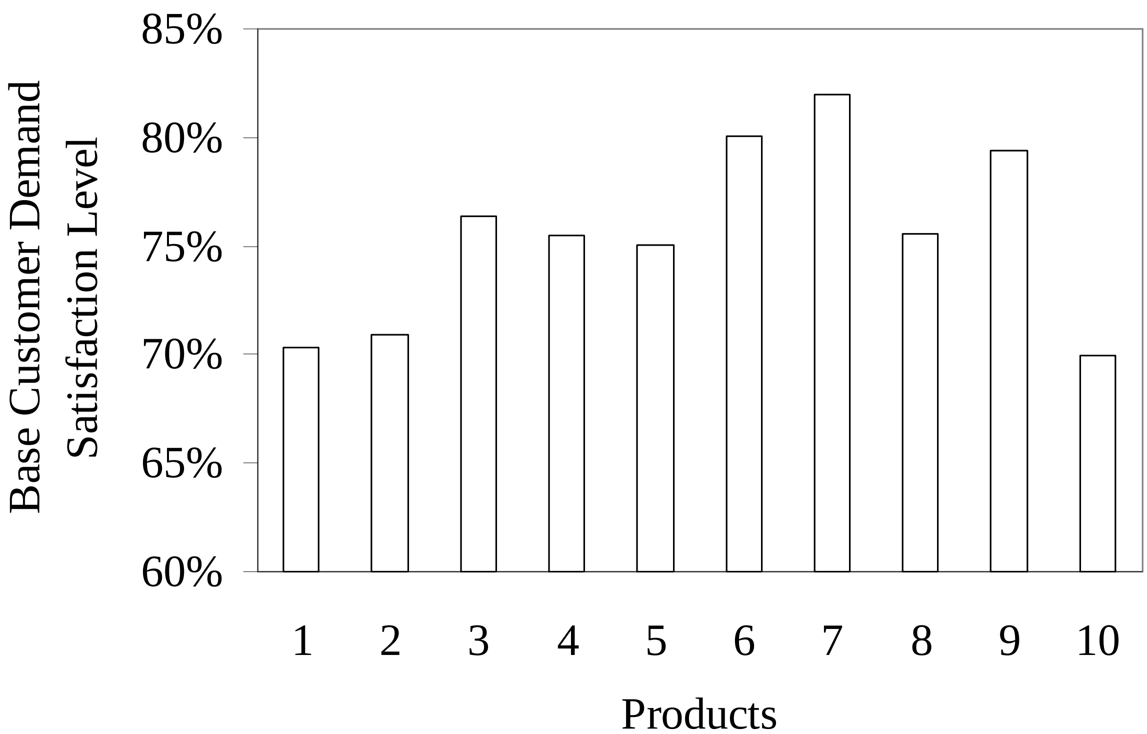


Figure 2. Base customer demand satisfaction levels in the supply chain

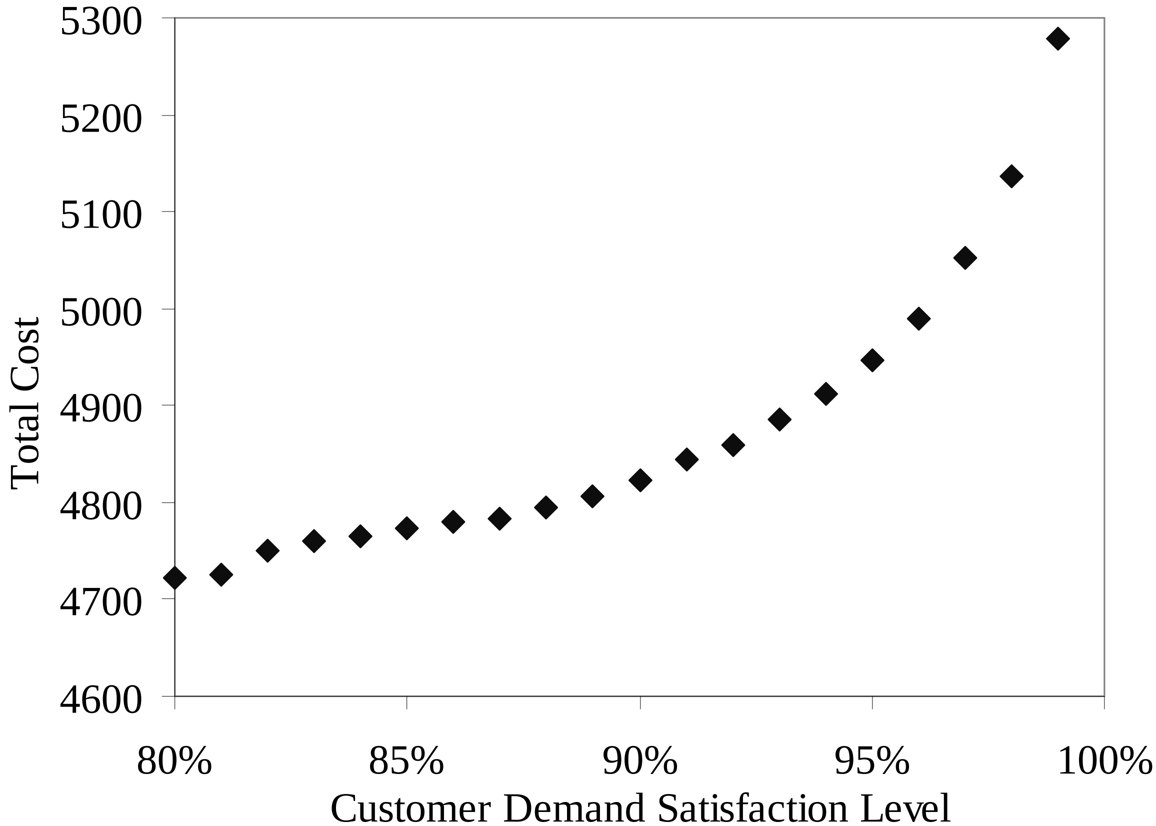


Figure 3. Variation of total cost with customer demand satisfaction level (α)

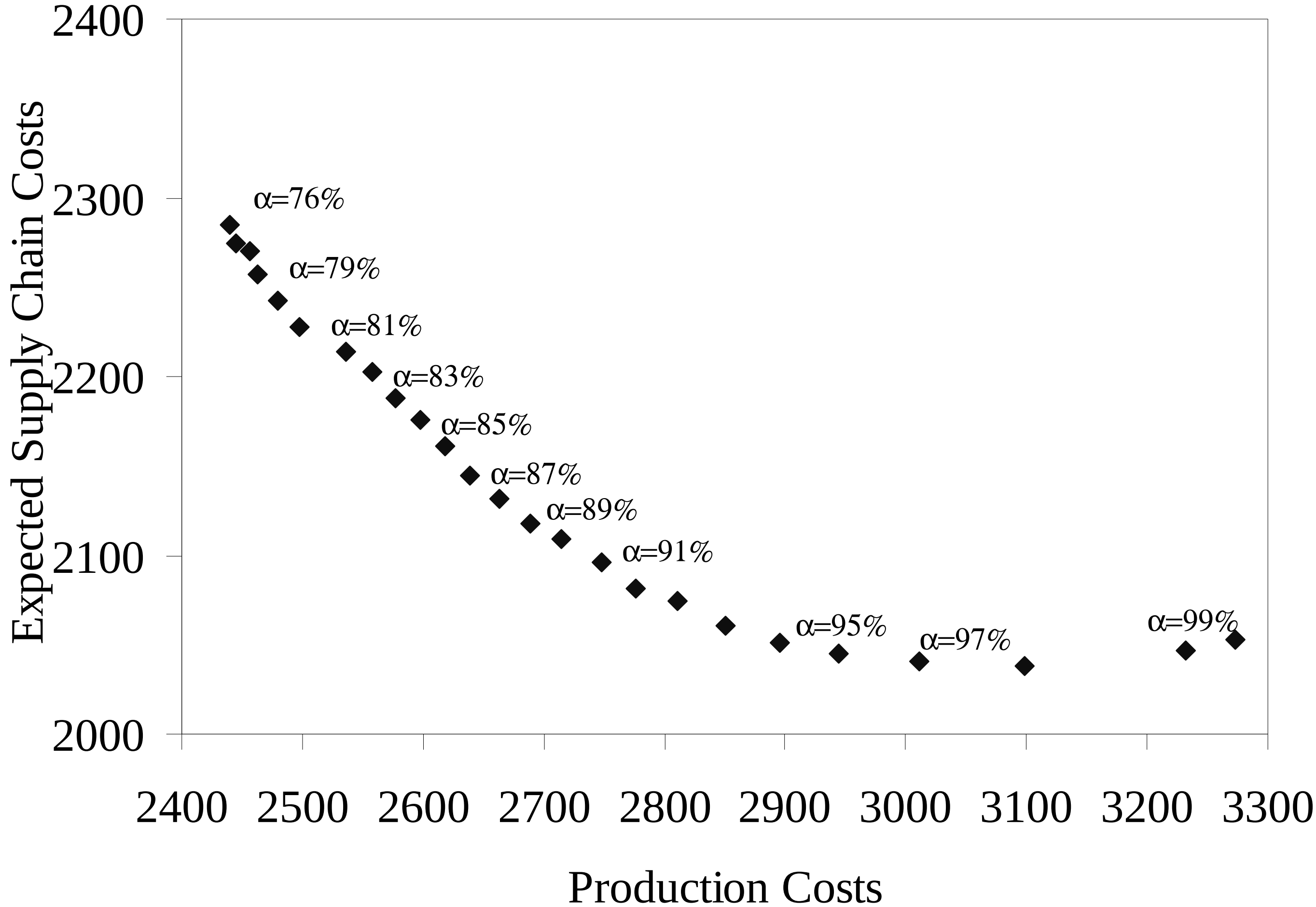
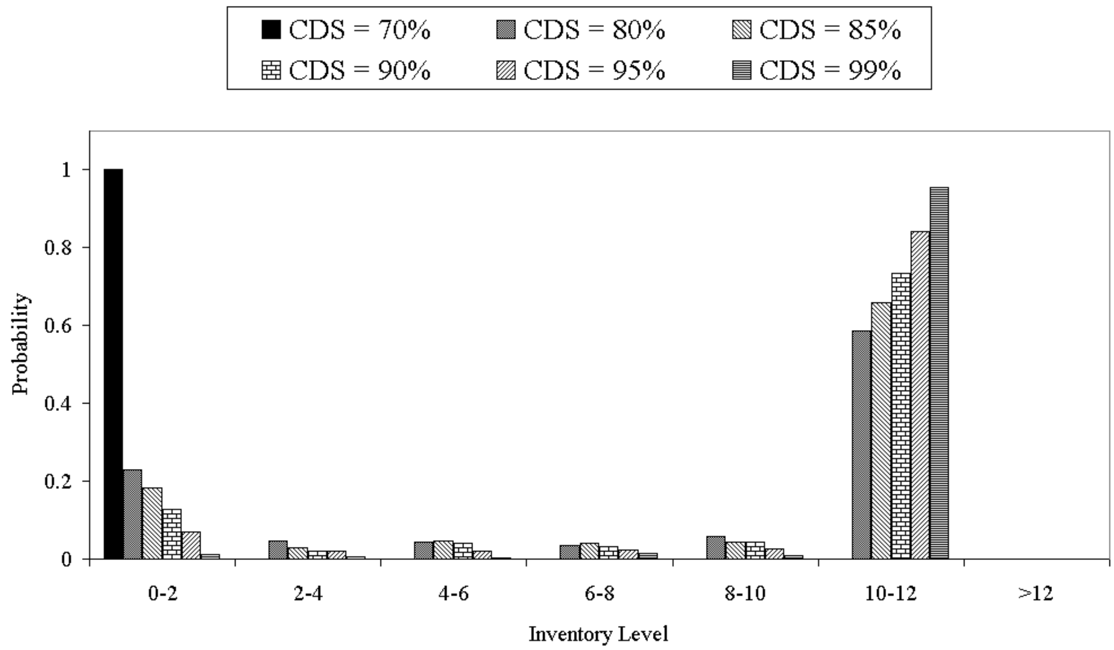
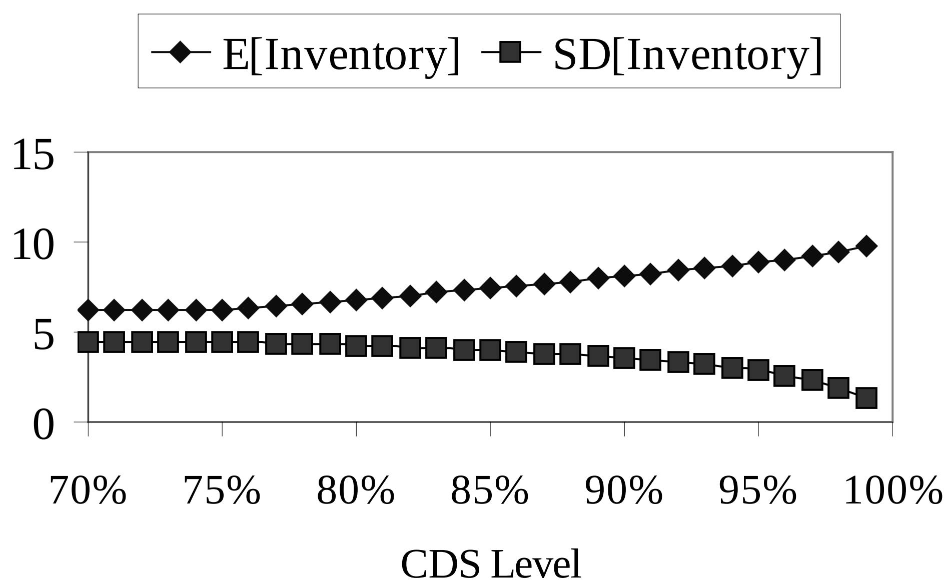


Figure 4. Trade-off curve between expected supply chain costs and production costs

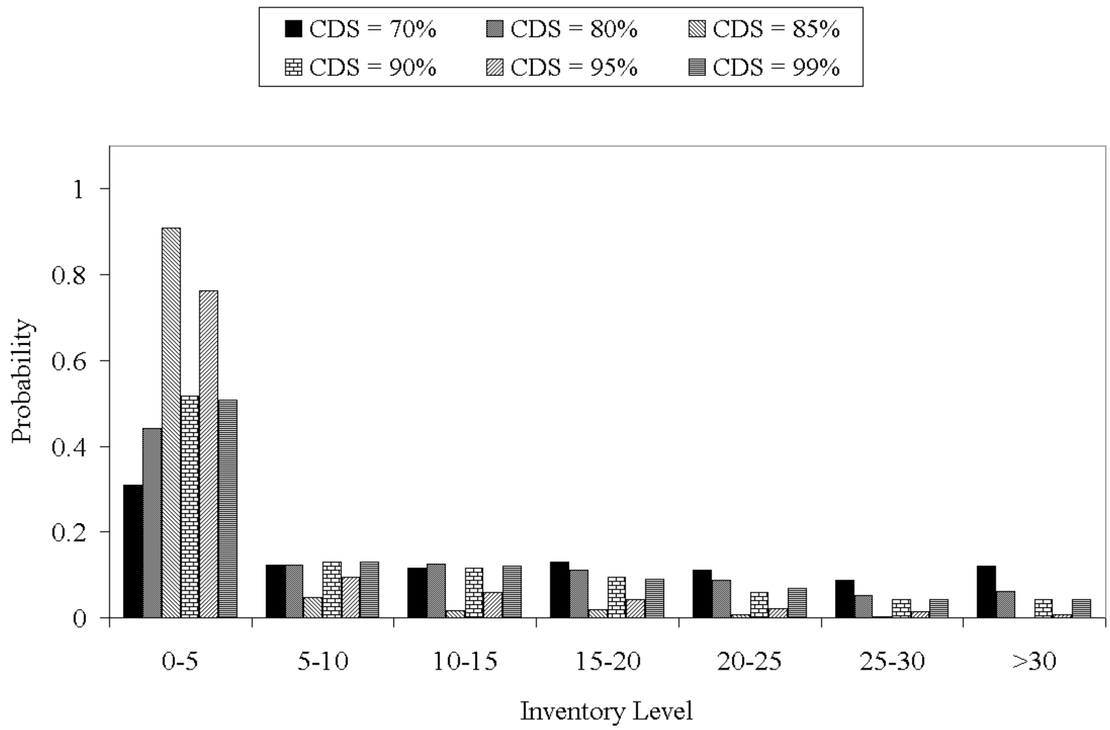


(a)

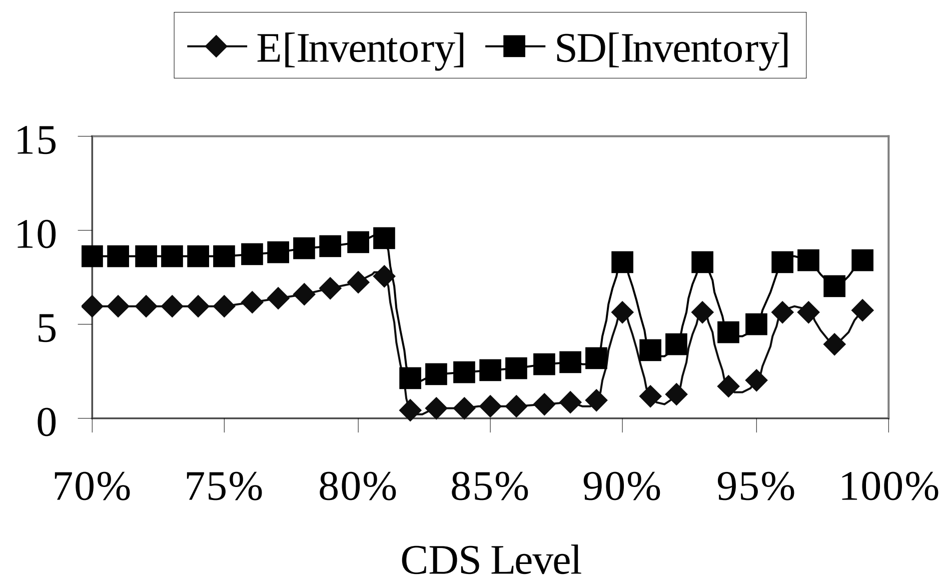


(b)

Figure 5. Variation of (a) probability distribution and (b) mean and standard deviation of inventory of product 1 at site 1 with CDS level



(a)



(b)

Figure 6. Variation of (a) probability distribution and (b) mean and standard deviation of inventory of product 1 at site 2 with CDS level

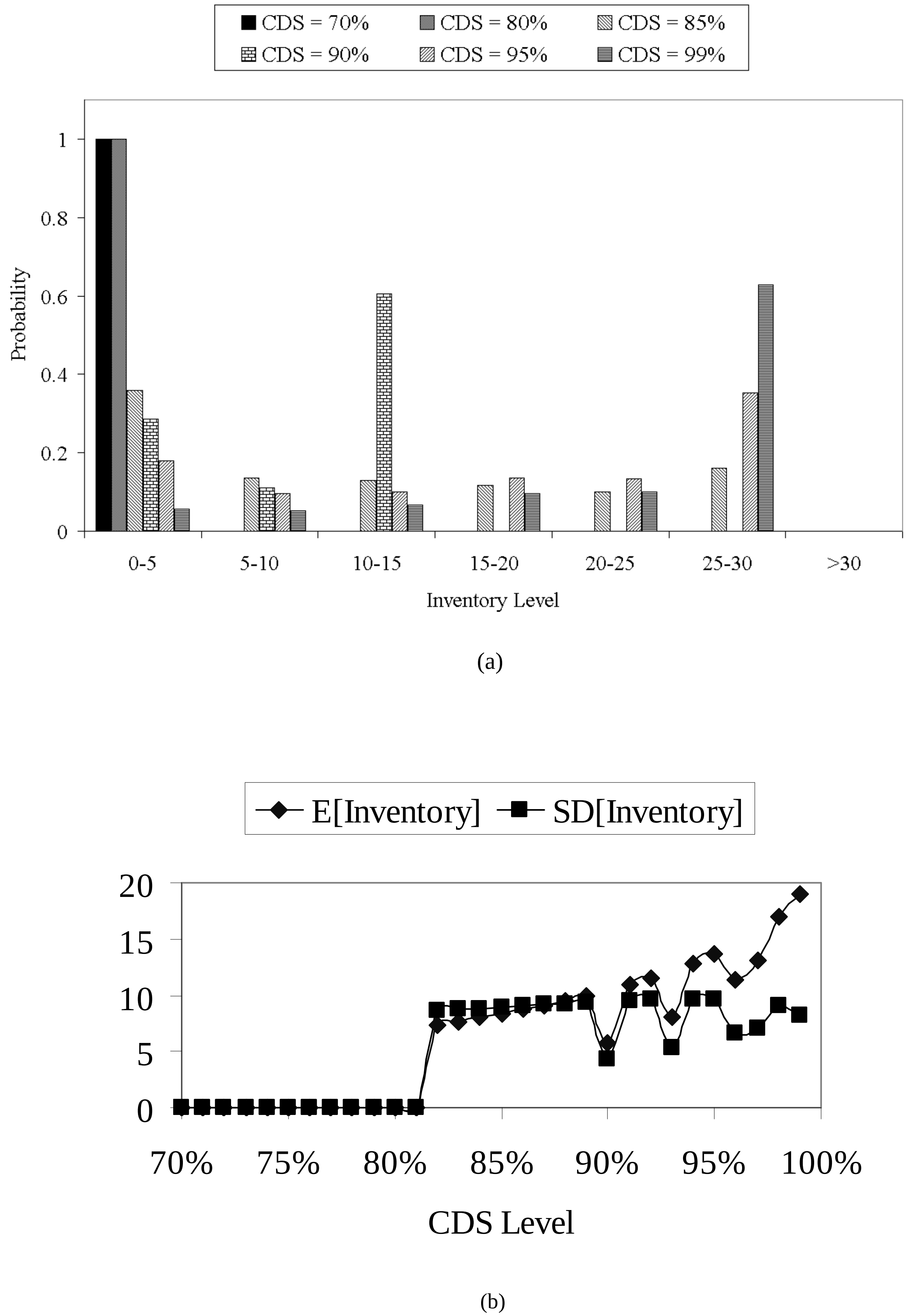


Figure 7. Variation of (a) probability distribution and (b) mean and standard deviation of inventory of product 1 at site 3 with CDS level

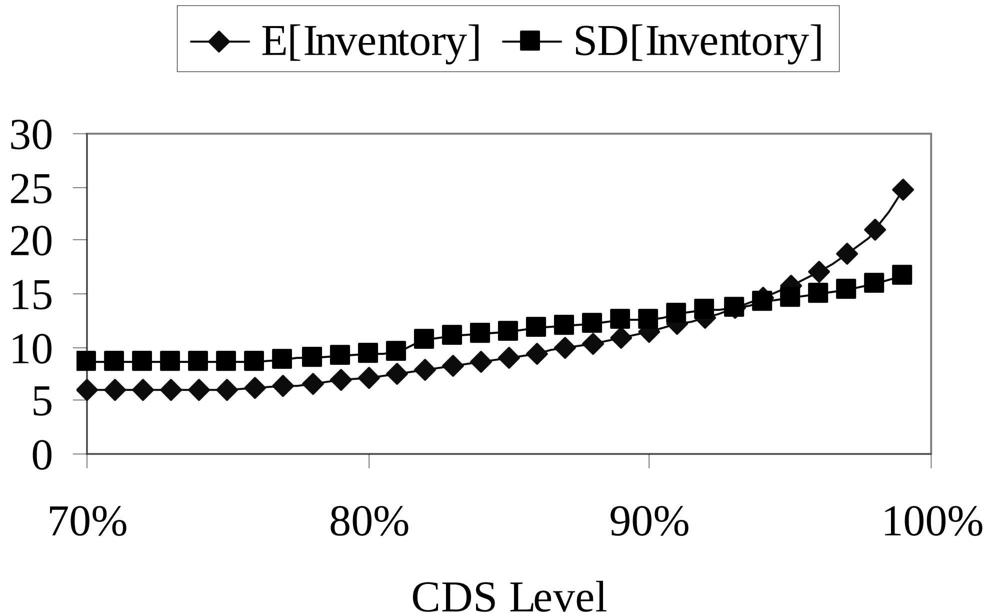


Figure 8. Hedging inventory risk at sites 2 and 3 by capacity integration

List of Captions