Multiperiod Planning of Multisite Supply Chains Under Demand Uncertainty

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In this work, the multiperiod planning of multisite supply chains under demand uncertainty is addressed. The presence of nested optimization problems (one for each period) coupled with the NP-hard nature of the original deterministic problem makes the multiperiod problem computationally intractable. Consequently, a modeling framework which incorporates partial information about the future demand evolution process is proposed for reducing the computational complexity of the multiperiod model. Specifically, the multiperiod model is reduced to an augmented two-stage model by considering the demand as uncertain in only the upcoming period in which the planning decisions need to be implemented immediately while treating it as deterministic in the remaining planning horizon. The coupling between the upcoming period and the rest of the planning horizon, which exists due to the transfer of inventory over time, is modeled by utilizing analytically derived expressions for the expected inventory. In addition, various rolling horizon planning policies incorporating different levels of future information are studied within a simulation environment.

1. INTRODUCTION

Uncertainty in product demand has been extensively studied in the process systems literature [1–3]. The variability in product demand can be traced back to the basic need of planning models which is to optimally allocate resources for the future using currently available information [4]. However, one of the key component not studied in great detail is the effectiveness of various rolling horizon planning policies, which would have to be implemented in a real multiperiod planning setting. In view of this, a model formulation which utilizes partial information about future uncertain demand is proposed for the deterministic planning model of McDonald and Karimi [5]. This methodology is based on the previous works of the authors for the single period case [3,6]. The main part of the paper investigates the quantitative impact of various alternative rolling horizon planning policies through a supply chain planning case study.

2. AUGMENTED TWO-STAGE FORMULATION

The proposed augmented two-stage formulation for the multiperiod planning model of McDonald and Karimi [5] (MP^{A2S}) is as follows.

$$\begin{aligned}\min \quad & \sum_{f,j,s,t} FC_{fst}Y_{fjst} + \sum_{i,j,s,t} \nu_{ijst}P_{ijst} + \sum_{i,s,t} p_{ist}C_{ist} + \sum_{i,s,s',t} t_{iss't}W_{iss't} \\ & + \sum_{i,t=1} \mathcal{E}_{\theta_{it}}\left[\mathcal{Q}_{it}(A_{ist},\theta_{it})\right] \\ & + \sum_{i,s,t\geq 2} t_{ist}S_{ist} + \sum_{i,s,t\geq 2} h_{ist}I_{ist} + \sum_{i,s,t\geq 2} \zeta_{ist}I^{\Delta}_{ist} + \sum_{i,s,t\geq 2} \mu_{it}I^{-}_{it}\end{aligned}$$

subject to

$$P_{ijst} = R_{ijst}RL_{ijst} \;\; ; \;\; C_{ist} = \sum_{i'} \beta_{i'is} \sum_{j} P_{i'jst} \;\; ; \;\; C_{ist} = \sum_{s'} W_{is'st} \tag{1}$$

$$FRL_{fjst} = \sum_{i:\lambda_{if}=1} RL_{ijst} \;\; ; \;\; \sum_{f} FRL_{fjst} \leq H_{jst} \tag{2}$$

$$MRL_{fjst}Y_{fjst} \leq FRL_{fjst} \leq H_{jst}Y_{fjst} \tag{3}$$

$$A_{is(t=1)} = I^0_{is} + \sum_{j} P_{ijs(t=1)} - \sum_{s'} W_{iss'(t=1)} \tag{4}$$

$$I_{is(t=2)} = \mathcal{E}_{\theta_{i(t=1)}}\left[I_{is(t=1)}\right] + \sum_{j} P_{ijs(t=2)} - \sum_{s'} W_{iss'(t=2)} - \sum_{s} S_{is(t=2)} \tag{5}$$

$$I_{ist} = I_{is(t-1)} + \sum_{j} P_{ijst} - \sum_{s'} W_{iss't} - \sum_{s} S_{ist} \tag{6}$$

$$\sum_{s} S_{ist} \leq \bar{\theta}_{it} \;\; ; \;\; \bar{\theta}_{it} - \sum_{s} S_{ist} \leq I^{-}_{it} \leq \bar{\theta}_{it} \;\; ; \;\; I^{L}_{ist} - I_{ist} \leq I^{\Delta}_{ist} \leq I^{L}_{ist} \tag{7}$$

$$\mathcal{Q}_{it}(A_{ist},\theta_{it}) = \left[\begin{array}{ll} \min & \sum_{i,s} t_{ist}S_{ist} + \sum_{i,s} h_{ist}I_{ist} \sum_{i,s} \zeta_{ist}I^{\Delta}_{ist} + \sum_{i} \mu_{it}I^{-}_{it} \\ \text{s.t.} & \\ & \sum_{s} S_{ist} \leq \theta_{it} \;\; ; \;\; I_{ist} = A_{ist} - S_{ist} \\ & \theta_{it} - \sum_{s} S_{ist} \leq I^{-}_{it} \leq \theta_{it} \;\; ; \;\; I^{L}_{ist} - I_{ist} \leq I^{\Delta}_{ist} \leq I^{L}_{ist} \end{array}\right] \tag{8}$$

The objective function of model MP^{A2S} is composed of three distinct components. The first component comprising of the first four terms accounts for the production costs incurred in the entire planning horizon. The corresponding production decisions, P_{ijst} (production amount), RL_{ijst} (runlength), C_{ist} (raw material consumption), $W_{is'st}$ (intersite shipment), Y_{fjst} (setup) and A_{ist} (availability), are constrained through the production constraints given by Equations 1 through 4. The second component (fifth term in objective function) captures the expected recourse costs for the first period as determined by the solution of the embedded inventory management optimization problem given by Equation 8. Finally, the last four terms model the supply chain costs incurred for $t \geq 2$.

The demand is modeled as deterministic for $t \geq 2$ and the supply chain decisions consisting of S_{ist} (supply), I_{ist} (inventory), I^{Δ}_{ist} (safety stock deficit) and I^{-}_{it} (customer shortage) are subject to Equation 7 which are the supply chain constraints. Note that in these constraints, the expected demand ($\bar{\theta}_{it}$) is used. The linking between the first time period and the remaining planning horizon is modeled through Equation 5 with the *expected* inventory level at the end of the period 1 providing the initial inventory for period 2.

Solution of model MP^{A2S} requires estimation of (i) the expected recourse function and (ii) the expected inventory levels for the first period. Exact deterministic equivalents for these two quantities are obtained by utilizing the analysis previously proposed by the authors [3,6]. The basic idea of the solution methodology consists of solving the recourse problem in Equation 8 analytically using linear programming duality followed by analytical integration for expectation evaluation [3]. The resulting optimal supply policies thus uncovered are subsequently used for determining the expected inventory level [6].

3. MULTIPERIOD PLANNING POLICIES

Model MP^{A2S} is *anticipative* in nature as it determines the production plan for the entire planning horizon prior to the actual demand realization for even the first period. The optimal production decisions for $t \geq 2$ are of no practical significance as they do not have to be implemented immediately. This inherent flexibility of modifying future production plans in response to the demand evolution process is studied within a simulation environment which captures the evolution of time and the corresponding resolution of uncertainty through various rolling horizon planning policies. These policies incorporate varying levels of information about the demand process while considering different future timespans for making current decisions. The policies studied are: (i) Single Period Deterministic (SPD) (ii) Single Period Stochastic (SPS) (iii) Multiperiod Deterministic (MPD) and (iv) Multiperiod Stochastic (MPS) planning.

In the SPD planning policy a planner solves single period deterministic planning problems within a rolling horizon setting. Aside from taking a very myopic view of the future by only considering the upcoming period, the planner also fails to recognize the uncertainty in the demand as captured by its standard deviation. After solving the appropriate model and fixing the resulting optimal production plan in the supply chain, random demand realizations are revealed to the planner. Based on these realizations the planner determines the optimal supply policies for the various production sites and the resulting total cost incurred. Even though the demand is considered to be deterministic by the SPD planner, the two-stage decision making framework is still recognized within which the supply chain variables can be optimally set to optimize in the face of uncertainty. The optimal supply policies translate into the post-demand satisfaction inventory levels and provide the planner with the initial conditions for the second period. The planner carries out this planning procedure in a rolling horizon manner for the entire planning horizon. This sequence of steps is then repeated to average over the randomly generated demand realizations.

The SPS policy is similar to the SPD policy with respect to the restrictive view taken of the future timespan. Unlike the SPD planner, however, the SPS planner has information about both the mean and the standard deviation of the demand. The SPS planner, thus, solves the single period stochastic formulation for determining the production setting for the upcoming period. Subsequently, demand realizations (same as those revealed for the SPD planner) are revealed to the SPS planner and the optimal supply chain decisions are made. Decisions are then made sequentially in a rolling horizon manner in the same spirit as the SPD case.

The third planning protocol considered is the MPD planning policy. In this policy, the view of the future is expanded to include the entire future timespan starting with the upcoming period while considering the demand to be deterministically known. The inventory transfer between time periods is described by deterministic inventory balance constraints of the form given by Equation 6. The optimal production plan is implemented in *only* the upcoming period thus retaining the flexibility to alter the production settings in the future in response to unfolding events. Finally, the MPS planner extends the MPD planning framework by characterizing the upcoming period demand by both its mean and standard deviation. The demand in the future periods, however, is still considered to be deterministic. The expected inventory level is used to link the stochastic period to the future deterministic periods through Equation 5.

To benchmark the quantitative performance of each of these planning strategies, the hypothetical Perfect Future Information (PFI) planning policy is used. The PFI planner is assumed to have complete information about the sequence of demands realized in the future. The PFI planner, thus, plans for the entire time horizon simultaneously by solving the deterministic multiperiod problem based on the randomly generated demand scenarios for each of the periods. This planning procedure results in *minimum* total costs in the supply chain as it is equivalent to considering all the decisions (both production and supply chain) as second stage, control decisions. Note that this policy, though, does not result in an implementable plan as the production decisions cannot be postponed to after demand realization. The optimal cost obtained, however, can be utilized to assess and compare the effectiveness of each of the planning policies.

4. EXAMPLE

The proposed solution methodology is applied to the supply chain planning example originally studied in Gupta, Maranas and McDonald [6]. The supply chain network consists of three production sites manufacturing ten products grouped into five product families. The demand for the products exists at a single customer over a planning horizon of six months corresponding to six planning periods of one month duration each. Each product family is manufactured on a single, limited capacity processing equipment and fixed setup charges are incurred for each production campaign. The product demands are assumed to be normally distributed with specified means and standard deviations [6].

First, the multiperiod deterministic MILP problem is solved assuming mean product demand using CPLEX accessed via GAMS resulting in an optimal cost of \$23,485. Next, the PFI policy is implemented within the example setting. The expected cost incurred through this policy is \$23,599. This provides the lower bound on the costs of the other planning policies. Note that the cost incurred by the PFI policy is only marginally higher than the deterministic cost. This is a result of the (unattainable) assumption of perfect future knowledge which is the basis of the PFI policy. Subsequently, the other four planning policies are simulated. The resulting expected multiperiod costs obtained are \$26,079 (SPD); \$25,341 (SPS); \$25,269 (MPD) and \$24,725 (MPS).

To quantify the performance of the proposed methodology, the *uncertainty gap reduction (UGR)* metric is defined. This is given by

$$\%\mathrm{UGR} = \left(\frac{z\,(\mathrm{MPD}) - z\,(\mathrm{MPS})}{z\,(\mathrm{MPD}) - z\,(\mathrm{PFI})} \right) \times 100 \tag{9}$$

where and $z\,(\cdot)$ represents the expected cost incurred through a particular planning policy. The denominator in Equation 9 represents the *uncertainty gap* which arises due to the failure of the MPD planner to account for uncertainty in the planning decisions. Equivalently, it is the amount the MPD planner would be willing to pay in return for complete information about the future demand realizations for the *entire* planning horizon. Similarly, the numerator in Equation 9 is the value allocated to information regarding the standard deviation of the upcoming period by the MPD planner. The fractional savings in cost achieved by switching from the MPD policy to the MPS policy are hence captured through the UGR metric.

For the current example setting, a UGR of 32.6% is obtained. This implies that the uncertainty gap can be reduced by almost a third by just incorporating an uncertain description of demand for the upcoming period. To gain further insights into the cost savings achieved through the MPS policy, the expected multiperiod costs incurred through the MPS and the MPD policies are analyzed in terms of their constitutive components. The results of this activity-based cost analysis are shown in Figure 1. As Figure 1 indicates, the MPS and the MPD policies result in almost comparable setup, transportation and customer shortage charges. Relatively insignificant difference in the fixed production charges implies that no additional setups are enforced by the MPS planner. Comparable transportation and customer shortage charges translate to comparable customer service levels achieved through the two planning policies.

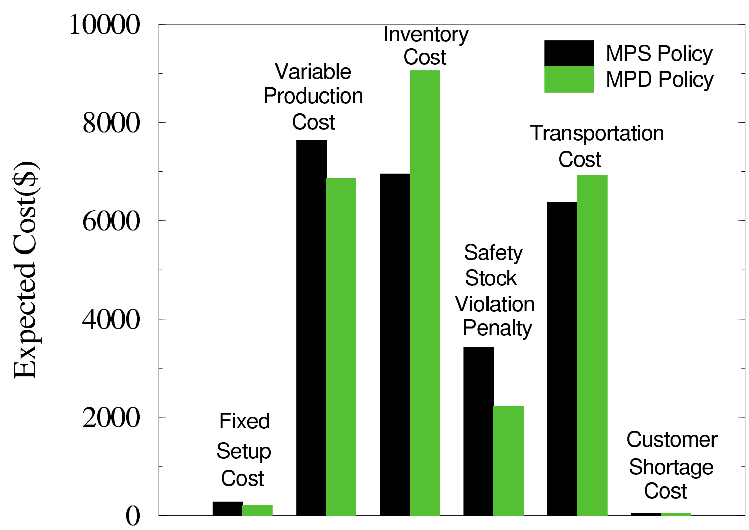

Figure 1. Activity-based cost analysis

The MPS policy, however, outperforms the MPD policy in terms of the inventory holding charges while the MPD policy results in lower variable production and safety stock violation charges. These observed cost trends may be intuitively interpreted as follows. The upcoming period demand can be satisfied by either (i) production in the upcoming period or (ii) inventory carryover. The MPS planner favors (relatively) the former strategy while the MPD planner relies primarily on the later. The production plans generated by the MPS policy can be expected to be more cost effective than the corresponding MPD policy plans for meeting the demand in the upcoming period. This can be attributed to the explicit incorporation of the variability in the upcoming period demand, in terms of its standard deviation, into the planning decisions through the augmented two-stage model in the MPS policy. Thus, higher production charges are incurred in return for lower inventory holding charges. Due to less reliance on inventory stock to meet demand, which translates into lower inventory levels, safety stock violations are frequent leading to high violation penalties. The MPD planner, on the other hand, does not incorporate any demand variability information while determining the production plan for the upcoming period. Consequently, use of inventory stock is preferred to meet customer demand resulting in lower production and safety stock violation charges with correspondingly high inventory holding charges. On the whole, the additional production and safety stock violation charges incurred are offset by the savings in inventory charges through the MPS policy leading to overall savings in the supply chain.

5. SUMMARY

In this work, the multiperiod planning of multisite supply chains was addressed. An augmented two-stage model, which utilizes partial information about the future uncertain demand, was formulated and subsequently embedded within a rolling horizon simulation framework. The fact that significant cost savings could be realized with the proposed methodology over deterministic planning policies was highlighted by a supply chain planning case study.

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