Real-Options-Based Planning Strategies under Uncertainty

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In this work, a real-options-based valuation (ROV) framework for hedging under uncertainty is developed. The basic idea of the proposed ROV methodology is the recognition and utilization of external market opportunities to guide internal planning decisions of a company. This is achieved by applying key financial planning principles such as arbitrage-free pricing and risk-neutral valuation to real, nonfinancial resource allocation decisions under uncertainty. Three application areas of the ROV methodology are described: production planning, pharmaceutical pipeline management, and emission trading. Multistage stochastic programming is used to incorporate uncertainty and a quantitative comparison of the ROV approach, and the traditional net-present-value (NPV) approach is provided.

Introduction

Continuous change, uncertainty, and intense competitive interactions are the defining characteristics of today’s volatile business environment. As a consequence of this continuous shifting of market expectations and customer loyalties, most investment decisions, ranging from the short-term, operational decisions to the long-term strategic decisions, have to be made under highly uncertain conditions. The uncertainty in most planning initiatives can essentially be traced back to the basic paradigm underlying these activities, which is the optimal allocation of future resources on the basis of current information and future predictions. This dependence on future predictions is the basic cause of all uncertainty because different managers, drawing on different past experiences and different sets of information, can come up with very different forecasts for the very same planning horizon. However, in the long run, the final measure of value under uncertainty is determined solely by the valuation mechanisms embedded within the financial markets. In view of this observation, we introduce a versatile modeling and solution framework for aligning the internal planning decisions of a company under uncertainty with the external financial markets.

Even though many important contributions have been made in the past few years, most of the research in the process systems engineering literature has focused within an enterprise’s boundaries without recognizing the existence of financial markets. Some of the settings that have been investigated include midterm production planning, multi-echelon supply-chain design, batch plant design and operation, and flexibility/reliability analysis. One of the key assumptions that is implicitly incorporated in all of the literature cited above is an adaptive posture toward combating uncertainty. This stance corresponds to exploiting the flexibility inherent in most planning settings through decision postponement in the face of uncertainty as modeled by a two-stage/multistage stochastic programming approach. Consequently, within this framework, no attempt is made to explicitly influence the underlying sources of uncertainty. Alternatively, a company might also adopt a more aggressive shaper position by attempting to restructure the uncertainty levels in order to limit downside risk while maintaining upside potential. Operationally, this can be achieved through special contracting agreements with the customer such as minimum quantity commitments in return for price discounts. In addition to such privately negotiated contracts, market-traded financial instruments, such as futures and options contracts, can also be used to manage the risk exposure of a company’s assets in accordance with its risk-bearing preference/capacity. In light of these observations, integration of a shaper attitude toward risk, as captured by the use of financial instruments, within an adaptive planning framework is introduced in this paper.

The remainder of the paper is organized as follows. In the following section, the limitations of the traditional net-present-value (NPV) approach to decision making under uncertainty are discussed with the aid of a simplified planning example. Subsequently, an economically corrected version of the NPV analysis is described in the form of the real-options-valuation (ROV) approach. Multiperiod production planning under demand uncertainty is then used as a benchmark setting for quantitatively contrasting the NPV and ROV approaches, and applications of the approach to pharmaceutical pipeline management and emissions trading are highlighted.

Limitations of the NPV Approach under Uncertainty

Traditionally, most resource allocation decisions have been guided by the discounted-cash-flow approach, which uses the classic NPV criterion to choose between alternative investment decisions. Furthermore, the basic deterministic NPV analysis can be extended to account for uncertainty and managerial flexibility through the decision-tree-analysis (DTA) framework, which maps out all alternative future actions contingent on all possible future states of nature. As a representative illustration of how the NPV analysis is typically incorporated within a DTA framework, consider a two-period planning horizon in which the production decision has to be made in the current period ($t = 0$) to meet an uncertain demand in the future period ($t = 1$). The
product demand, which is assumed to follow a binomial distribution, is known for the current period. In the following period, the demand is expected to either increase or decrease with given probability values. The uncertainty in demand is translated into the uncertainty in the value realized by the company (Figure 1) through the following simple supply-chain model

\[
V_u = \mu \times \min(P, u\theta_0) \quad (1)
\]

\[
V_d = \mu \times \min(P, d\theta_0) \quad (2)
\]

The above expressions determine the future revenue streams contingent on the demand realized in the future period \((u\theta_0 \text{ or } d\theta_0)\), the production decision \((P)\) made in the current period, and the unit sales price for the product \((\mu)\). Subsequently, by discounting the expected future revenue flows using an expected rate of return, the NPV \((V_0)\) is calculated and utilized to establish the optimal production amount through the solution of the following optimization model

\[
\max_{p>0} V_0 = \frac{1}{1 + \tilde{r}} [pV_u + (1 - p)V_d] \quad (3)
\]

The primary challenge in using the above-described methodology is the estimation of the expected rate of return \((r)\) used for translating future cash flows into their present value equivalents. Basic finance theory defines \(r\) as the equilibrium expected rate of return on securities equivalent in risk to the project being valued.\(^{29}\)

The two most popular means for determining this key parameter have been through (i) an ad hoc classification of risk and (ii) use of the capital asset pricing model (CAPM).\(^{30}\)

The former approach relies on defining a risk spectrum in terms of a qualitative description of risk (such as low, medium, high), followed by the allocation of a specified rate of return to each of these risk categories. Subsequently, the rate of return is determined and used for valuation based on the perceived location of the project being valued within this risk spectrum. As one would expect, this approach exposes the valuation procedure to a significant amount of managerial subjectivity and can thus be used to provide only a rough, initial estimate of the value of a project.

The CAPM, which is the most widely used approach, resolves the subjectivity that arises with the risk classification methodology by utilizing the variance as the appropriate risk metric.\(^{30–32}\) Specifically, the rate of return is given by

\[
r = \tilde{r} + \beta [E(r_m) - \tilde{r}] \quad (4)
\]

where

\[
\beta = \frac{\text{Cov}(\tilde{r}, r_m)}{\text{Var}(r_m)} \quad (5)
\]

In the above relations, \(r\) is the risk-free rate of return; \(\tilde{r}\) is the return on a financially traded twin security that is perfectly correlated with the project being valued; \(r_m\) is the rate of return of the entire market; and \(E(\cdot)\), \(\text{Cov}(\cdot, \cdot)\), and \(\text{Var}(\cdot)\) are the expectation, covariance, and variance operators, respectively. The task of obtaining the rate of return under the CAPM is thus reduced to that of identifying the twin security and its corresponding \(\beta\) value for the calculation of the risk premium \((\beta[E(r_m) - \tilde{r}]\).

Even though the CAPM has been widely applied for making capital budgeting decisions, several key limitations of this approach can be identified. First, the model is based on the assumption that the variance in returns is the appropriate risk metric for distinguishing between competing projects. Because variance is a symmetric measure, it fails to recognize the basic asymmetric risk preference of most companies/individuals. Second, there is an implicit dependency between the tasks of identifying the twin security and determining the value of the project that is being tracked by that security. For instance, consider the simple supply-chain model of eq 3. To identify the perfectly correlated twin security, the future payoffs \(V_u\) and \(V_d\) need to be known. However, these values depend on the optimal production level \(P\) through eqs 1 and 2, which, in turn, cannot be determined unless the twin security (and subsequently the rate of return) is known. These limitations of the NPV approach motivate the adoption of an alternative valuation framework that makes use of real-options-based valuation (ROV).

Real-Options-Valuation (ROV) Approach. The key idea behind the ROV technique is the extension of the theory developed for financial options\(^{33,34}\) to real, nontraded assets.\(^{35}\) The basic observations on which this approach is based are as follows: (1) It is typically easier to track the underlying source of uncertainty rather than the effect of uncertainty through market-traded instruments. (2) The value of a project can be tracked more effectively through a portfolio of securities rather than a single security. (3) A tracking portfolio can be periodically rebalanced in accordance with the changing risk characteristics of the project.

As an illustrative example of how the ROV methodology incorporates the above-described features, this approach is applied to the simplified supply-chain planning setting introduced in the previous section. In view of this objective, suppose that the product being manufactured is market-traded, implying that there are efficient and liquid spot and futures markets in which this product can be bought and sold. Currently, markets such as the New York Mercantile Exchange (www.nymex.com) and The Chicago Board of Trade (www.cbot.com) exist for several products spanning industrial settings ranging from energy (crude oil, heating oil, natural gas) to metals (gold, copper, silver) and agriculture (soybean, wheat, corn). Such market settings effectively translate the balance between the forces of supply and demand into a clear price signal. For example, if the aggregate demand for the product is higher (lower) than the aggregate supply, then the price of a futures contract written on that product is expected to increase (decrease). Thus, a futures contract can be used as a twin security for tracking demand uncertainty.
The solution of eqs 6 and 7 determines the composition of the replicating portfolio as given by

\[ N = \frac{V_u - V_d}{S_u - S_d} = \frac{V_u - V_d}{uS_0 - dS_0} \]  

(8)

\[ B = \frac{1}{1 + r_f} \left( \frac{S_u V_u - S_d V_d}{S_u - S_d} \right) = \frac{1}{1 + r_f} \left( \frac{dV_u - uV_d}{u - d} \right) \]  

(9)

Using eqs 8 and 9, the present value of the replicating portfolio is

\[ NS_0 - B = \frac{1}{1 + r_f} \left[ (1 + r_f - d) V_u + (u - 1 - r_f) V_d \right] \]

\[ = \frac{1}{1 + r_f} [qV_u + (1 - q)V_d] \]

(10)

where

\[ q = \frac{1 + r_f - d}{u - d} \]

(11)

At this point, the fundamental arbitrage-free pricing principle is invoked, which states that, if two assets have the same payoff in all future states, then they must be identically priced. If this were not the case, then market participants could potentially make risk-free profits by simultaneously buying the undervalued asset and selling the overvalued one. Basic finance theory suggests that such arbitrage opportunities do not exist in efficient, transparent markets. In light of this principle, the present value of the uncertain future revenue streams is, thus, the same as the present value of the replicating portfolio, implying that

\[ V_0 = \frac{1}{1 + r_f} [qV_u + (1 - q)V_d] \]

(12)

Comparison of eq 12 with eq 3 highlights the two basic differences between the NPV and ROV approaches. First, NPV analysis uses the expected rate of return (r), whereas the ROV analysis uses the risk-free rate of return (r_f) for discounting future cash flows. Second, NPV analysis determines the expected value of future cash flows using the true probability (p) of demand variability, as opposed to the risk-neutral probability (q) used by the ROV approach.

**Supply-Chain Planning.** The midterm planning model proposed by McDonald and Karimi is first used as a benchmark to illustrate the ROV methodology. This model has the underlying structure of the capacitated lot-sizing problem. Details regarding the deterministic formulation can be found in McDonald and Karimi and Gupta and Maranas. Classification of the variables and constraints of this model into production- and supply-chain-type categories results in a two-stage decision-making framework that is utilized to incorporate demand uncertainty for the single period setting.

Extension of the single-period framework to the multi-period case results in the model formulation described in Appendix A.

In the context of the multiperiod production planning (MPP) model, the difference between the ROV and NPV analyses arises in the rate of return used for discounting and the probability distribution used for application of the expectation operator \( E_{\alpha, \beta}(\cdot) \). As described in the previous section, NPV uses the risk-free rate of return.
in conjunction with the risk-neutral probabilities, whereas ROV uses the objective probabilities and the expected rate of return. The demand uncertainty is modeled through a discrete, multiplicative binomial process. In such a setting, the expected rate of return given by

\[ r = \frac{pS_u + (1-p)S_d}{S_0} - 1 = pu + (1-p)d - 1 \quad (13) \]

is used in the NPV approach. A representative planning case study is described next to illustrate how model MPP can result in significantly different planning results within the ROV and NPV frameworks.

A manufacturing enterprise that produces a single product is planning its production activities over a planning horizon of 10 time periods. The parameters characterizing this operation are listed in Table 1. On the basis of the cost estimates available to the company, a profit margin of approximately 25% is forecasted. As indicated in Table 1, the available production capacity is well in excess of the current product demand. Future market forecasts for the product are bullish, with an expected demand growth rate of approximately 19%.

The MPP model is solved for ROV and NPV using the CPLEX 7.0 solver accessed via GAMS for the data given in Table 1. For the ROV approach, the risk-free rate is assumed to be 5%, resulting in a risk-neutral probability of 0.53. The optimal expected costs obtained for the two alternative valuation approaches are listed in Table 2, along with their breakdowns in terms of the various constitutive components. As indicated by these results, expected cost savings of approximately 42% are forecasted by the adoption of an ROV-based planning approach over the traditional NPV approach. The cost analysis presented in Table 2 also provides valuable insights into the sources of these observed savings. In particular, these savings are primarily attributed to a 3-fold reduction in customer shortage penalties.

Figure 5 shows the temporal variation of cost over the planning horizon, indicating that consistent cost savings are realized throughout the planning horizon by the ROV approach. The magnitude of these savings, however, varies significantly from period to period (Figure 6), ranging from a low of 8% to a high of almost 60%. In addition to expected cost, the temporal profiles of the expected capacity utilization (Figure 7), inventory level (Figure 8), and customer shortage (Figure 9) are examined. Almost identical profiles are obtained (see Figure 7), with capacity utilization ranging from 40 to 95% over the planning horizon. The increase in capacity utilization over time is attributed to the expected growth in future demand. On the logistics side, significant reduction in expected inventory is predicted (see Figure 8).
Figure 9 demonstrates that the reduction in inventory level does not come at the expense of compromising the customer service level. Customer demand satisfaction of 100% is achieved for the first half of the planning horizon, whereas a significant portion of the demand is lost as a result of capacity limitations in the second half. The fractions of missed sales, however, are almost identical for the ROV and NPV approaches.

The seemingly inconsistent result that the ROV approach leads to a 3-fold reduction in customer shortage penalty (as indicated in Table 2) while maintaining the same customer service level (as shown in Figure 9) can be explained from a hedging portfolio viewpoint. Using the risk-neutral probability in conjunction with the risk-free rate of return to evaluate the recourse function in model MPP is equivalent to setting up a portfolio consisting of a financial asset (the twin security) and a real asset (the supply-chain operation). Consequently, given the perfect correlation between the demand realized and the value of the twin security, the decrease in value of the real asset in the face of high demand (because of capacity limitations) is offset by the increase in value of the financial asset. This also explains why the major portion of the savings is concentrated in the later half of the planning horizon as seen in Figure 5. Thus, a more stable, robust cost profile is obtained in the face of demand uncertainty using the ROV analysis.

**Pharmaceutical Pipeline Management.** The current growth in the pharmaceutical sector has largely been fueled by new-product pipelines and the promise of novel drugs in the future. These product pipelines are in a constant state of flux as new drug leads are identified and products reach the market or are discontinued during development because of safety/efficacy concerns. As a result of this dynamic market state, the optimal management of the new-product pipeline has moved to the forefront of all strategic planning initiatives of a pharmaceutical company.

Every drug in the pharmaceutical pipeline undergoes a well-defined development process consisting of a number of distinct, sequential stages. After the drug discovery process in which the drug lead is identified, optimized, and tested in animals, the drug is taken through three phases of clinical testing. Phase I studies are aimed at determining the toxicity level of the drug and are usually carried out in small populations of patients. Following the successful completion of phase I trials, phase II trials are undertaken in which the pharmacokinetic and pharmacodynamic characteristics of the drug are determined. Finally, large-scale phase III trials are conducted to establish the effectiveness of the drug. This is achieved by comparing the therapeutic potential of the drug with the performance of an existing treatment. Once sufficient evidence regarding the safety and efficacy of the drug is collected, an investigational new drug application (IND) is filed with the Food and Drug Administration (FDA). Approval of the IND culminates in the commercialization and large-scale production of the drug.

The entire drug development process is characterized by significant technical and market uncertainty. Negative test results such as excessive toxicity and/or unforeseen side effects for a given drug candidate terminate any further development. Even at the IND approval stage, chances of failure are significant in light of ever-changing and ever-tightening regulatory restrictions. On the market side, incomplete information regarding the cost of producing the drug, the pricing structure, and the market share that can be captured translate into significant uncertainty in the drug's market value. The cumulative impact of these uncertain conditions is further amplified by the relatively long duration (8–10 years) and the large investment ($500–$800 million) required for the development process. Accounting for uncertainty is thus critical in effectively managing a company’s drug development activities. In recognition of this fact, we address the pharmaceutical pipeline management problem as a portfolio optimization problem within the ROV framework.

The problem is defined as follows: Given a set of candidate drugs in various stages of development; estimates of the probability of success, duration of testing, and investment required for the remaining stages; and forecasts for the future market values, determine the optimal drug development portfolio that maximizes the ROV.

The key idea that distinguishes the proposed ROV approach from the substantial contributions to new-product development in the process systems engineering community is that explicit tracking of the uncertainty in the market value of the drug through market-traded securities. This can be achieved by constructing a portfolio of securities whose value is correlated with the market value of the drug under development. For instance, suppose that the drug under consideration is a cancer drug; then, the market value of this drug can be reasonably tracked with a portfolio of biotechnology companies specializing in developing cancer treatments. The drug development process can then be viewed as a compound real option on the value of this portfolio. A detailed model description and results can be found in the recent works of Rogers et al.

The proposed methodology is highlighted by applying it to a pharmaceutical pipeline consisting of five drugs in various stages of development, as depicted in Figure 10. Drugs 1 and 2 have just completed phase I trials, drug 3 is awaiting application of the IND to the FDA, and drugs 4 and 5 are ready to be taken into large-scale phase III trials. A schematic of the drug development process is shown in Figure 11. This diagram depicts the unfolding of technical and market uncertainties over time and the planning decision points throughout the time horizon. Specifically, the key decision that is modeled is the abandonment option that is available at the end of every stage of the development process as...
indicated in Figure 11. This abandonment option derives its value from the underlying market uncertainty by limiting the downside risk under unfavorable market conditions (as captured by low market values). The individual valuations for the five drugs are presented in Figure 12. The results shown in Figure 12 demonstrate the value of the abandonment option, especially for drugs 1 and 2, for which this option is worth more than twice the original (without-abandonment) value. In addition to the value of the drug, the optimal abandonment schedule is also obtained, as shown in Figure 13 for drug 2, where phase II trials for drug 2 are undertaken given the current market-value estimate, $MV_0$. At the end of phase II, if the market value is in excess of $3.2MV_0$, then phase III trials are conducted; otherwise, the abandonment option is exercised. Subsequently, upon successful completion of phase III trials, a market value in excess of $MV_0$ is required for initiation of the IND approval phase with the FDA. Optimization of the entire pharmaceutical pipeline subject to budgetary restrictions yields the optimal product portfolio consisting of drugs 1–3 and 5. The decision to not pursue development of drug 4 is attributed to the lack of sufficient budgetary resources at the start of the planning horizon. The value of the ROV approach lies in providing external market discipline to internal product development decisions.

**Emissions Trading.** In the past decade, market-based environmental policy has emerged as a more cost-effective alternative to the conventional “command-and-control” standards of environmental law and regulation.44,45 Emission markets operate by issuing tradeable permits denominated in units of a specific pollutant (e.g., pounds or tons of SO2) in amounts equivalent to their allowable emissions over a given period of time (e.g., 1 year). All permits are transferable, so that, if a facility can generate excess permits by reducing emissions below its allotted amounts, then it can sell these extra permits to other facilities. At regular intervals, facilities submit emission reports for the compliance period, which can range anywhere from 3 months to 1 year. At that time, facilities must own sufficient permits to cover emissions to avoid a noncompliance penalty charged by the regulatory authority for any excess emissions. Having been used to cover emissions, these permits are then retired from the regulatory compliance system, preventing subsequent use or transfer.

The cost efficiency achieved in meeting environmental targets through a market-based approach, however, comes at the expense of exposing the enterprise to market volatility, in terms of both the number of such permits available and their price. These uncertainties can be traced back to the variability in emission levels that arises because of uncertainty in the demand for a company’s goods/services, variability in the quality of fuel and other raw materials consumed, and randomness in weather and other environmental factors.46 Consequently, failure to recognize emission uncertainty and the resulting market volatility can pose severe financial, operational, and political challenges.

In addition to emission permits, market-priced option contracts written on these permits can also be used for compliance purposes. Options are contracts that give the holder the right, but not the obligation, to purchase an emission permit at a specified price (known as the strike price) and time for a one-time, upfront premium payment. The key feature of an option is its asymmetrical payoff. Because the contract does not imply any obligation to buy the permit, the holder of the contract profits from favorable price changes while being protected from adverse ones. In the spirit of the real-options approach, a global compliance portfolio view is adopted for pollution abatement planning by formulating the decision problem as follows:

Given a set of candidate technologies characterized by their respective emission levels, fixed capital investments and variable production costs, current market prices, and the availability of emission permits and emission options, along with future demand and market forecasts, determine the optimal technology—permits—
options compliance portfolio that minimizes the total expected cost. A multistage stochastic programming approach is used to incorporate demand and market uncertainties. Representative results for a particular case study are presented next, whereas a detailed treatment of the problem can be found in Gupta and Maranas. A manufacturing enterprise that is planning its pollution abatement activities has six potential technology candidates under consideration. These six technologies span the entire spectrum of cost-emission possibilities ranging from the low-cost–high-emission extreme to the high-cost–low-emission alternative. Emission permits are currently available at a 40% discount over the noncompliance penalty, and the future permit price is expected to lie anywhere between 0 and 100% of the noncompliance penalty with equal probability. In addition to the permits, option contracts are also available. The premium payment for an option increases as the strike price decreases, reflecting the willingness of the company to pay a higher premium initially in exchange for obtaining the future right to purchase an emission permit at a lower cost. Three alternative settings are investigated according to whether permits and/or options are included in the compliance portfolio. The cost distributions predicted are shown in Figure 14. Significant cost savings are achieved by including emission permits and options in the compliance portfolio. Specifically, the optimal technology and permits portfolio (T + P) outperforms the purely technology based portfolio (T) by 19% with respect to total cost savings. The flexibility provided by the options contracts translates into an additional savings of 4% over the T + P portfolio. The risk profile is also altered favorably through the inclusion of permits and options, as indicated by the reduction in the probability of excessively high cost scenarios in Figure 14 for the T + P and T + P + O portfolios. The savings forecasted can be primarily attributed to the reduction in noncompliance penalties. These expected savings in noncompliance charges can be traced back to the reduction of the excess emissions. Figure 15 indicates that the probability of adequately meeting emission requirements is increased from a low of roughly 10% with technology only to around 95% through the inclusion of permits and options.

Summary

In this work, a new real-options-based approach to valuation under uncertainty was proposed. The key managerial insight underlying this methodology was that external financial market information can be used for quantifying and evaluating internal planning decisions of an enterprise. In particular, this was achieved by taking a global portfolio management perspective on planning initiatives and recognizing that both financial and real assets can be used to achieve desired objectives. This amounted to taking a more aggressive shaper attitude toward planning under uncertainty in contrast to a purely passive adaptive one.

From a quantitative viewpoint, the ROV analysis was obtained through a natural extension of the theory developed for financial options. The fact that it is easier to track the underlying source of uncertainty rather than the effect of uncertainty in financial markets was exploited through the replication portfolio approach. Subsequently, in light of the arbitrage-free pricing principle, the risk-neutral pricing approach under uncertainty was uncovered. Multistage stochastic programming was identified as the framework within which this approach could be embedded to address various planning settings.

The benefits of the proposed methodology were highlighted through three seemingly unrelated planning examples. The first case study, supply-chain planning under demand uncertainty, contrasted the ROV approach with the traditional NPV approach in terms of both model formulation and the resulting business implications. The second, pharmaceutical pipeline management, case study showcased how market discipline can be applied to high-risk internal activities such as new-product development. Finally, the emissions trading setting emphasized the general applicability of the portfolio management view that forms the basis of the ROV analysis.

The three case studies presented spanned the entire “distance-from-market” spectrum. The distance from market of a particular setting refers to the degree to which the underlying source of uncertainty of that setting is tracked in financial markets. The supply-chain planning example was closest to the market because demand uncertainty for a wide range of products can be efficiently tracked in well-developed, highly liquid futures markets. Pharmaceutical pipeline management constituted the other extreme of this spectrum, given the limited number of market-traded securities through which the market value of a novel drug can be tracked. Emissions trading is intermediate between the other two cases given that emission markets, though reasonably efficient and liquid, are still not as well established as commodity markets. We believe that, in the future, the potential advantages of adopting an ROV methodol-
ogy will be increased as additional risks are unbundled and securitized in free-market settings.

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Appendix A. Multiperiod Midterm Planning Model

Sets

\[ \mathcal{I} = \{t\} = \text{set of time periods} \]

Parameters

- \( c^{\text{set}} \): fixed setup cost.
- \( c^{\text{var}} \): variable production cost.
- \( c^{\text{tran}} \): transportation cost.
- \( h \): inventory holding cost.
- \( \mu \): customer shortage penalty.
- \( \theta_t \): demand in period \( t \).
- \( P_t \): production capacity.
- \( I^0 \): initial inventory.

Variables

- \( Y_t \): 1, if product is made in period \( t \); 0, otherwise.
- \( P_t \): production level in period \( t \).
- \( S_t \): supply to customer in period \( t \).
- \( S^-_t \): customer shortage in period \( t \).
- \( I_t \): inventory level at the end of period \( t \).

Utilizing the above-described notation, the multiperiod production planning model (MPP) is formulated as follows.

\[
\text{MPP} \quad \min \, c^{\text{set}} Y(t_0) + c^{\text{var}} P(t_0) + c^{\text{tran}} S(t_0) + \mu S^-(t_0) + h I(t_0) + \frac{1}{1+r} \mathcal{O}(t_0)(S(t_0), I(t_0), \theta(t_0))
\]

subject to

\[
P(t_0) \leq \bar{P} Y(t_0)
\]

\[
S(t_0) \leq \theta(t_0)
\]

\[
I(t_0) = I^0 + P(t_0) - S(t_0)
\]

\[
S^-(t_0) = \theta(t_0) - S(t_0)
\]

\[
P(t_0), S(t_0), I(t_0), S^-(t_0) \geq 0, \quad Y(t_0) \in \{0, 1\}
\]

where

\[
\mathcal{O}(S(t_0), I(t_0), \theta(t_0)) = \min \left[ c^{\text{set}} Y(t_1) + c^{\text{var}} P(t_1) + c^{\text{tran}} S(t_1) + \mu S^-(t_1) + h I(t_1) + \frac{1}{1+r}\mathcal{O}(t_1)\right]
\]

subject to

\[
P(t_1) \leq \bar{P} Y(t_1)
\]

\[
S(t_1) \leq \theta(t_1) + S^-(t_1)
\]

The first stage of model MPP corresponds to the planning process for the current time period \( t = 0 \), for which the demand is assumed to be known with certainty. The first five terms in the objective function account for the fixed setup costs and the variable production, transportation, customer shortage, and inventory holding costs, respectively, incurred in the current period. These costs are minimized subject to the production capacity (eq 14), customer supply (eq 15), inventory balance (eq 16), and customer shortage (eq 17) constraints. The costs incurred in the future time periods \( t \geq 1 \) are captured in the objective function through the recourse function \( Q_t(S_t, I_t, \theta_{t+1}) \). This function, which consists of a number of nested optimization problems as shown in eq 18, accounts for the nonanticipative resolution of demand uncertainty over a multiperiod planning horizon in a recursive manner. The linking across consecutive time periods arises through the transfer of inventory and customer shortages. This results in the propagation of uncertainty through the planning horizon.

Literature Cited


